Day 307 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan(\theta - 20°) = \sqrt{3}$
for $-180° \leq \theta \leq 180°$.

Let $u = \theta - 20°$; then $\tan u = \sqrt{3}$
Base solutions: $u = 60°$ (+ multiples of 180°)
$\theta = (u + 20°) \div 1$, keeping those in range
Answer: $\theta = -100°, 80°$

2 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{x + 2} = x - 4$.

Square both sides: $x + 2 = (x - 4)^2$.
$x^2 - 9x + 14 = 0$, so $x = 2$ or $x = 7$.
Check in $\sqrt{x + 2} = x - 4$: $x = 7$ gives $3 \geq 0$ (valid).
$x = 2$ gives right-hand side $-2 < 0$, so it is rejected.
Answer: $x = 7$

3 Force up a rough slope: find weight and friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight W N is on a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A force P N acts up a line of greatest slope; the particle stays at rest for $10.8 \leq P \leq 25.2$.
Find (a) W, (b) the coefficient of friction, (c) the acceleration when $P = 50.4$. Take $g = 9.8$ m s^{-2}.

Limiting friction acts down the plane at $P = 25.2$ and up the plane at $P = 10.8$
$P_{max} = W\sin\alpha + F$ and $P_{min} = W\sin\alpha - F$, where $F = \mu W\cos\alpha$

Adding: $2W\sin\alpha = 36$, so $W = \dfrac{36}{2 \times \frac{3}{5}} = 30$

Subtracting: $2F = 14.4$, so $F = 7.2 = \mu \times 30 \times \frac{4}{5}$, giving $\mu = 0.3$

$P = 50.4$: net force up the plane $= 50.4 - 25.2 = 25.2$, and mass $= \frac{30}{9.8}$

Answer: (a) 30 N (b) 0.3 (c) 8.23 m s^{-2}

4 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 4 per week. After a modification, 2 faults are found in 1 week.
Test at the 5% level whether the rate of faults has decreased.

H_0: $\lambda = 4$ per week, H_1: $\lambda < 4$ per week
Over 1 week: $X \sim Po(4)$
$P(X \leq 2) = 0.238$
$0.238 > 5\%$, so do not reject H_0
There is insufficient evidence that the rate has decreased
Answer: Do not reject H_0 (p-value 0.238)

5 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 11x + 28 \geq 0$ and $3x - 8 < 10$.

Quadratic: $(x - 4)(x - 7) \geq 0$ gives $x \leq 4$ or $x \geq 7$.
Linear: $3x - 8 < 10$ gives $x < 6$.
Both must hold: take the overlap of the two sets.
Answer: $x \leq 4$

Day 307 **A Level Maths: S Tier 10-a-day: Model Solutions**

6 Particle on a smooth slope joined to hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (10 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a smooth slope, $\sin\alpha = \frac{3}{5}$, and B is 3.75 m above the ground.

Released from rest, find the further distance A moves up the slope after B lands (A does not reach the pulley).

A: $T - 6 \times 9.8 \times \frac{3}{5} = 6a$. B: $10 \times 9.8 - T = 10a$

Adding: $9.8(10 - 6 \times \frac{3}{5}) = 16a$, so $a = 3.92$

$T = 10(9.8 - 3.92) = 58.8$

When B lands: $v^2 = 2 \times 3.92 \times 3.75 = 29.4$

Then A decelerates at $g\sin\alpha = 5.88$: further distance $= \frac{29.4}{2 \times 5.88} = 2.5$

Answer: 2.5 m

7 Trigonometric integrals with identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use a suitable trigonometric identity to find the exact value of

$\int_0^{\pi/6} \sin^4 3x \, dx$

Use $\sin^2 A$ or $\cos^2 A$ twice: write as $(\ldots)^2$ with the double angle formula
Integrate term by term
Substitute the limits

Answer: $\frac{1}{16}\pi$

8 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(9 - 5x)^{-\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(9 - 5x)^{-\frac{3}{2}} = 9^{-\frac{3}{2}}(1 - \frac{5}{9}x)^{-\frac{3}{2}} = \frac{1}{27}(1 + u)^{-\frac{3}{2}}$, $u = -\frac{5}{9}x$.

$(1 + u)^{-\frac{3}{2}} = 1 + (-\frac{3}{2})u + \frac{15}{8}u^2 + (-\frac{35}{16})u^3 + \ldots$

Substitute u and multiply by $\frac{1}{27}$.

Valid when $|u| < 1$, i.e. $|x| < \frac{9}{5}$.

Answer: $\frac{1}{27} + \frac{5}{162}x + \frac{125}{5832}x^2 + \frac{4375}{314928}x^3 + \ldots$; $|x| < \frac{9}{5}$

9 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (−3, 0, 3), (−1, 0, −1), (−2, 5, 3).
Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\text{BA} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix}$; $\text{BA} \cdot \text{BC} = 18$

$|\text{BA}| = \sqrt{20}$, $|\text{BC}| = \sqrt{42}$

$\cos \text{ABC} = \frac{18}{\sqrt{20}\sqrt{42}} = 0.6211$, so $\text{ABC} = 51.6°$

$|\text{BA} \times \text{BC}| = \left|\begin{pmatrix} -20 \\ 4 \\ -10 \end{pmatrix}\right| = \sqrt{516}$

Area $= \frac{1}{2}|\text{BA} \times \text{BC}|$

Answer: ABC = 51.6°, area = $\sqrt{129}$

10 Normal test: critical region and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, 64)$. H_0: $\mu = 100$ is tested against H_1: $\mu > 100$ using the mean of a sample of size 25 at the 1% level.

(a) Find the critical region for the sample mean.

(b) Find the probability of a Type II error when $\mu = 103$.

Under H_0, sample mean ~ $N(100, \frac{64}{25})$, standard error = 1.600

$z = 2.3263$ for the upper tail: critical value = $100 + 2.3263 \times 1.600 = 103.7$

Type II error: do not reject H_0, sample mean < 103.7, where mean ~ N(103, 1.600)

$z = \frac{103.7 - 103}{1.600} = 0.451$

Probability = 0.674

Answer: (a) sample mean > 104 (b) 0.674