Day 302 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-2x-6| < |3x-6|$.

Square both sides: $(-2x-6)^2 < (3x-6)^2$.
$(-5x)(x-12) < 0$.
Critical values $x = 0$ and $x = 12$.
The quadratic has negative coefficient of x^2, so choose the outer region.
Answer: $x < 0$ or $x > 12$

2 Compound change with logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

£8000 is invested at 4% per year compound interest.
Find the least whole number of years for the investment to exceed £16000.

$8000 \times 1.04^n > 16000$, so $1.04^n > 2$
$n > \frac{\ln 2}{\ln 1.04} = 17.67$
n must be a whole number, so $n = 18$
Answer: 18

3 Transformations: new equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 5x + 2$ is translated by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

and then a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

Find the equation of the resulting curve in the form $y = ax^2 + bx + c$.

Translation by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$: $y = f(x-2) - 1 = x^2 - 9x + 15$.

Then a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis: replace x by 2x: $y = f(2x-2) - 1$.

Expand and simplify to give the quadratic.
Answer: $y = 4x^2 - 18x + 15$

4 Normal: unknown standard deviation then a second probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The lengths of a component, in mm are modelled by $N(85, \sigma^2)$. The probability of a value above 94 is 0.15.
(a) Find σ.
(b) Find the probability of a value above 104.

$P(X > 94) = 0.15$ so $Z = \frac{94-85}{\sigma} = 1.0364$

$\sigma = \frac{9}{1.0364} = 8.684$

$z = \frac{104-85}{8.684} = 2.188$

$P(X > 104) = P(Z > 2.188) = 0.0143$
Answer: (a) 8.68 (b) 0.0143

5 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 600 kg along a horizontal road, with driving force 1470 N. Resistances are 600 N on the car and 450 N on the trailer.

(a) Find the tension in the tow bar. At 6 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{1470 - 600 - 450}{2100} = 0.2$

Trailer: $T - 450 = 600 \times 0.2$, so $T = 570$

After the break the trailer decelerates at $\frac{450}{600} = 0.75$, and stops after $t = \frac{6}{0.75} = 8$ s

Trailer travels $\frac{6}{2} \times 8 = 24$ m

Car: $a = \frac{1470 - 600}{1500} = 0.58$, so $s = 6 \times 8 + \frac{1}{2} \times 0.58 \times 8^2 = 66.56$ m

Distance apart = $66.56 - 24$

Answer: (a) 570 N (b) 42.56 m

6 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = \frac{24}{x^2 + 3}$ at the point where $x = 1$

meets the axes at A and B. Find the exact area of triangle OAB, where O is the origin.

At $x = 1$: $y = 6$. Differentiate: gradient $= -3$

Tangent: $y - 6 = (-3)(x - 1)$, so $y = -3x + 9$

y-intercept 9; x-intercept 3

Area $= \frac{1}{2} \times |9| \times |3|$

Answer: $\frac{27}{2}$

7 Inverse function and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The function f is defined by $f(x) = 2x^2 - 12x + 19$ for $x \geq k$. Given that k is the least value for which f is one-one, find k, $f^{-1}(x)$, and the domain of f^{-1}.

Complete the square: $f(x) = 2(x - 3)^2 + 1$.

The vertex is at $x = 3$, so $k = 3$ (f is one-one on one side of it).

Let $y = f(x)$: $y - 1 = 2(x - 3)^2$, so $(x - 3) = \pm\sqrt{\frac{y - 1}{2}}$.

As $x \geq 3$, take the positive root: $x = 3 + \sqrt{\frac{y - 1}{2}}$.

Range of f = domain of f^{-1}: $f(3) = 1$, so $x \geq 1$.

Answer: $\mathbf{k = 3;\ f^{-1}(x) = 3 + \sqrt{\frac{x - 1}{2}};\ x \geq 1}$

8 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent. Find x and y.

$P(A) = 4x + x = 5x$, $P(B) = x + y$, $P(A \cap B) = x$

Independent: $x = 5x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{5}$

Total probability: $4x + x + y + 0.35 = 1$, so $4x + (x + y) = 0.65$

$4x = 0.65 - \frac{1}{5} = 0.45$

$x = \frac{9}{80}$, $y = \frac{1}{5} - x = \frac{7}{80}$

Answer: $\mathbf{x = \frac{9}{80},\ y = \frac{7}{80}}$

9 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (5, −2, 1), (0, −2, −1), (4, 1, 5).
Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\text{BA} = \begin{pmatrix}5\\0\\2\end{pmatrix}$, $\text{BC} = \begin{pmatrix}4\\3\\6\end{pmatrix}$; $\text{BA} \cdot \text{BC} = 32$

$|\text{BA}| = \sqrt{29}$, $|\text{BC}| = \sqrt{61}$

$\cos \text{ABC} = \frac{32}{\sqrt{29}\sqrt{61}} = 0.7608$, so $\text{ABC} = 40.5°$

$|\text{BA} \times \text{BC}| = \left|\begin{pmatrix}-6\\-22\\15\end{pmatrix}\right| = \sqrt{745}$

$\text{Area} = \frac{1}{2}|\text{BA} \times \text{BC}|$

Answer: ABC = 40.5°, area = $\frac{1}{2}\sqrt{745}$

10 Closest approach of two ships (AQA, Edexcel, OCR, MEI, Eduqas)

Two ships A and B move with constant velocities. At time t = 0 hours, A is at $(\mathbf{i} - 6\mathbf{j})$ km relative to a fixed origin and B is at $(-14\mathbf{i} - 11\mathbf{j})$ km. A has velocity $(\mathbf{i} + 5\mathbf{j})$ km h^{-1} and B has velocity $(5\mathbf{i} + 8\mathbf{j})$ km h^{-1}.
Find (a) the time at which the ships are closest together, (b) the least distance between them.

Position of B relative to A: $\mathbf{d} = (-15\mathbf{i} - 5\mathbf{j}) + t(4\mathbf{i} + 3\mathbf{j})$

$|\mathbf{d}|^2 = (4t - 15)^2 + (3t - 5)^2 = 25t^2 - 150t + 250$

Least when $\frac{\text{d}}{\text{d}t}|\mathbf{d}|^2 = 0$: $t = -\frac{-75}{25} = 3$

$\mathbf{d} = (-15\mathbf{i} - 5\mathbf{j}) + 3(4\mathbf{i} + 3\mathbf{j}) = (-3\mathbf{i} + 4\mathbf{j})$, so least distance = 5

Answer: (a) t = 3 h (b) 5 km