Day 301 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The breaking strength of a cable is N(μ, 16) with known variance 16. The mean was claimed to be 30 kN. A random sample of 9 has mean 33.75. Test at the 5% level whether the mean is greater than 30.

H_0: $\mu = 30$, H_1: $\mu > 30$. Under H_0, the sample mean ~ $N(30, \frac{16}{9})$

$z = \frac{33.75 - 30}{4 \div \sqrt{9}} = 2.812$

p-value = $P(Z > 2.812) = 0.00246$

$0.00246 < 5\%$, so reject H_0

There is sufficient evidence at the 5% level that the mean has changed as claimed

Answer: Reject H_0 (z = 2.812, p-value 0.00246)

2 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 284 inclusive that are divisible by 3 or by 7, but not by both.

Multiples of 3: 94 terms (3 to 282): sum = 13395.
Multiples of 7: 40 terms: sum = 5740.
Multiples of both, i.e. of 21: 13 terms: sum = 1911.
Required sum = 13395 + 5740 − 2 × 1911.

Answer: 15313

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a point O on horizontal ground with speed 49 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{3}{4}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 29.4^2 - 2 \times 9.8 \times H$, so H = 44.1

At height $\frac{H}{2}$ = 22.05: $v_y^2 = 29.4^2 - 2 \times 9.8 \times 22.05 = 432.18$

v_x = 39.2 (constant), so $speed^2 = 1536.64 + 432.18 = 1968.82$

Answer: (a) 44.1 m (b) 44.4 m s^{-1}

4 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact gradient of the curve $y = 5^{x+1}$ at the point where x = 3.

$y = e^{(x+1)\ln 5}$

$dy/dx = 1 \ln 5 \times 5^{x+1}$

At x = 3: exponent = 4, so $dy/dx = 1 \ln 5 \times 5^4$

Answer: 625ln 5

5 Coded data: mean, standard deviation, Σx² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 observations x, $\Sigma(x - 60) = 10$ and $\Sigma(x - 60)^2 = 185$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 60) = \frac{10}{20} = 0.5$, so mean of x = 60.5

Variance = $\frac{185}{20} - (0.5)^2 = 9$

$\sigma = \sqrt{9}$

$\Sigma x^2 = \Sigma(x - 60)^2 + 120\,\Sigma(x - 60) + 20 \times 60^2$
$= 185 + 1200 + 72000$

Answer: (a) 60.5 (b) 3.00 (c) 73385

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6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = 2t^3 - 18t$.
Find the coordinates of the points on the curve where the gradient is -3.

$dx/dt = 4t$, $dy/dt = 6t^2 - 18$, so $dy/dx = \frac{6t^2 - 18}{4t}$

Set equal to -3: $6t^2 + 12t - 18 = 0$
$t = -3$ or $t = 1$
Answer: (18, 0) and (2, −16)

7 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(3r - 4) = 300$, find n.

$\sum_{r=1}^{n}(3r - 4) = 3 \times \frac{n(n+1)}{2} - 4n$.

$\frac{3\,n(n+1)}{2} - 4n = 300$, so $3n^2 - 5n - 600 = 0$.

Solving $(n > 0)$: $n = 15$.
Answer: n = 15

8 Projectile through a given point: two angles (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 30 m s^{-1}, at angle θ above the horizontal. It passes through the point P, 54 m horizontally and 29 m vertically from O.
Find the two possible values of $\tan\theta$, and the two possible values of θ to 1 d.p. Take g = 9.8 m s^{-2}.

Horizontal: $x = 30\cos\theta \times t$. Vertical: $y = 30\sin\theta \times t - 4.9t^2$

Eliminating t: $y = x\tan\theta - \frac{9.8x^2}{2u^2}(1 + \tan^2\theta)$

$\frac{9.8 \times 54^2}{2 \times 30^2} = 15.876$, so $29 = 54\tan\theta - 15.876(1 + \tan^2\theta)$

$15.876\tan^2\theta - 54\tan\theta + 44.876 = 0$

$\tan\theta = \frac{13}{9}$ or $\frac{863}{441}$, so $\theta = 55.3°$ or $62.9°$

Answer: $\tan\theta = \frac{13}{9}$ or $\frac{863}{441}$; $\theta = 55.3°$ or $62.9°$

9 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two boats P and Q move so that, t hours after noon, their position vectors in km are
$\mathbf{r}_P = \begin{pmatrix}1\\-5\end{pmatrix} + t\begin{pmatrix}0\\-2\end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix}-8\\10\end{pmatrix} + t\begin{pmatrix}3\\-5\end{pmatrix}$.
Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}-9\\15\end{pmatrix} + t\begin{pmatrix}3\\-3\end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix}3\\-3\end{pmatrix} = 0$

$-72 + t(18) = 0$, so $t = 4$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}3\\3\end{pmatrix}$, with length $\sqrt{18}$

Answer: t = 4, minimum distance = $3\sqrt{2}$ km

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10 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-39x^2 + 57x - 48}{(x + 4)(3x - 3)^2}$ in the form

$\frac{A}{(x + 4)} + \frac{B}{(3x - 3)} + \frac{C}{(3x - 3)^2}$.

$-39x^2 + 57x - 48 \equiv A(3x - 3)^2 + B(x + 4)(3x - 3) + C(x + 4)$.

Put $x = -4$: $-900 = A(-15)^2$, so $A = -4$.

Put $x = 1$: $-30 = C(5)$, so $C = -6$.

Compare x^2: $-39 = 9A + 3B$, so $B = -1$.

Answer: A = −4, B = −1, C = −6