Day 3 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos(2\theta + 40°) = \frac{\sqrt{2}}{2}$ for $-90° \leq \theta \leq 270°$.

Let $u = 2\theta + 40°$; then $\cos u = \frac{\sqrt{2}}{2}$

Base solutions: $u = 45, -45°$ (+ multiples of 360°)

$\theta = (u - 40°) \div 2$, keeping those in range

Answer: $\theta = -42.5°, 2.5°, 137.5°, 182.5°$

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x - 4$ and $g(x) = x^2 - 4$.

Solve $fg(x) = gf(x)$.

$fg(x) = 2(x^2 - 4) - 4 = 2x^2 - 12$.

$gf(x) = (2x - 4)^2 - 4 = 4x^2 - 16x + 12$.

Equate: $2x^2 - 16x + 24 = 0$.

Solve the quadratic: $x = 2$ or $x = 6$.

Answer: $x = 2$ or $x = 6$

3 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 18 and upper quartile 29. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 10 observations have mean 24. The largest is 47, an outlier, and it is removed. Find the new mean.

IQR = 29 − 18 = 11, so 1.5 × IQR = 16.5

Lower limit = 18 − 16.5 = 1.5, upper limit = 29 + 16.5 = 45.5

$\Sigma x = 10 \times 24 = 240$, so without 47: $\Sigma x = 193$

New mean = $\frac{193}{9}$

Answer: (a) 1.5 to 45.5 (b) 21.44

4 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 15 s, P is back at its starting point.

Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 5 × 12 + ½(12 − W)(5) − W(5) = 0

30 + 30 − 2.5W − 5W = 0, so 7.5W = 60

W = 8

v = 0 where the line from (5, 12) to (10, −8) crosses the axis: $t = 5 + 5 \times \frac{12}{20} = 8$

Greatest distance = area of triangle = ½ × 8 × 12

Answer: (a) W = 8 (b) 48 m

5 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of $\sin(\sin^{-1}(\frac{7}{25}) + \cos^{-1}(\frac{5}{13}))$.

Let $A = \sin^{-1}(\frac{7}{25})$ and $B = \cos^{-1}(\frac{5}{13})$, both acute

$\sin A = \frac{7}{25}$, $\cos A = \frac{24}{25}$, $\cos B = \frac{5}{13}$, $\sin B = \frac{12}{13}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{7}{25} \times \frac{5}{13} + \frac{24}{25} \times \frac{12}{13}$

Answer: $\frac{323}{325}$

6 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.8, P(B) = 0.15 and P(A | B) = 0.25.
(a) Find P(A ∩ B).
(b) Find P(A | B′).

P(A ∩ B) = P(A | B) × P(B) = 0.25 × 0.15 = 0.0375
P(A ∪ B) = 0.8 + 0.15 − 0.0375 = 0.9125
P(B′) = 0.85
P(A ∩ B′) = P(A) − P(A ∩ B) = 0.7625
$P(A \mid B') = \frac{0.7625}{0.85}$

Answer: (a) 0.0375 (b) $\frac{61}{68}$

7 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each year Jo pays £1000 into an account on 1 January.
Interest of 5% is added each 31 December.
Find the first year-end at which the balance exceeds £12,000.

After n years: $1000(1.05) + 1000(1.05^2) + \ldots + 1000(1.05^n)$.

GP: a = 1000 × 1.05, r = 1.05: $S_n = \frac{1000 \times 1.05\,(1.05^n - 1)}{0.05}$.

$S_n > 12000 \Rightarrow 1.05^n > 1.5714 \Rightarrow n > 9.264$.
So n = 10 years.
Answer: after 10 years

8 Stationary points: exact coordinates (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary point of
$y = \frac{\ln x}{x}$ for x > 0,
and determine its nature.

$dy/dx = \frac{1 - \ln x}{x^2} = 0$ gives $\ln x = \frac{1}{1}$

$y = e^{-1}$ when x = e
Second derivative at this point is negative, so it is a maximum
Answer: (e, e^{-1}), maximum

9 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3x^3 + 4x^2 - 36x - 33}{(x+3)(x-3)}$ in the form $ax + b + \frac{C}{(x+3)} + \frac{D}{(x-3)}$.

Divide: the numerator has degree 3, the denominator degree 2.
$(3x^3 + 4x^2 - 36x - 33) \div (x^2 - 9)$ gives quotient 3x + 4.
Remainder: −9x + 3.
Put x = −3: remainder = C(−6) so C = −5. Put x = 3: D = −4.
Answer: $3x + 4 - \frac{5}{(x+3)} - \frac{4}{(x-3)}$

10 Acceleration given, find when P returns to O (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t = 0, P passes through the fixed point O with velocity −80 m s^{-1}. At time t seconds, P has acceleration (−3t + 26) m s^{-2}.
Find (a) the times at which P is at O again, (b) the greatest distance of P from O before it first returns to O.

$v = \int a\,dt = -1.5t^2 + 26t - 80$ (v = −80 when t = 0)
$s = \int v\,dt = -0.5t^3 + 13t^2 - 80t$ (s = 0 when t = 0)
s = −0.5t(t − 10)(t − 16), so P is at O when t = 10 and t = 16
Greatest distance when v = 0: $-1.5t^2 + 26t - 80 = 0$, so t = 4 (the smaller root)
|s| = 0.5 × 4 × 6 × 12
Answer: (a) t = 10, t = 16 (b) 144 m