Day 296 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Scores are N(84, 9^2). The highest 5% and the lowest 5% of scores are discarded.
Find the range of scores that remain, correct to 3 significant figures.

The remaining middle 90% lies between z = ±1.6449
Lower limit = 84 − 1.6449 × 3 = 79.07
Upper limit = 84 + 1.6449 × 3 = 88.93
Answer: 79.1 to 88.9

2 Displacement of the form R sin(ωt + φ) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a line. Its displacement from a fixed point O at time t seconds is s metres, where s = 5 sin 2t + 12 cos 2t.
Find (a) the greatest distance of P from O, (b) the greatest speed of P, (c) the first time P is at its greatest distance from O (3 s.f.).

Write s = R sin(2t + φ) with R cos φ = 5 and R sin φ = 12

R = $\sqrt{(5^2 + 12^2)}$ = 13 and tan φ = $\frac{12}{5}$, so φ = 1.176 rad

Greatest distance = R = 13

v = $\frac{ds}{dt}$ = 13 × 2 cos(2t + φ), so greatest speed = 26

s is greatest when 2t + φ = $\frac{\pi}{2}$: t = (1.5708 − 1.176) ÷ 2

Answer: (a) 13 m (b) 26 m s^{-1} (c) 0.197 s

3 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = \frac{5}{2}$ and

$u_{n+1} = \frac{1}{1 - u_n}$.

Show that the sequence is periodic and find

$\sum_{r=1}^{99} u_r$.

$u_1 = \frac{5}{2}$, $u_2 = -\frac{2}{3}$, $u_3 = \frac{3}{5}$, $u_4 = \frac{5}{2}$.

$u_4 = u_1$, so the sequence repeats with period 3.

One period sums to $\frac{73}{30}$.

99 = 3 × 33 + 0: 33 full periods plus the first 0 terms.

Answer: $\frac{803}{10}$

4 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) lies on a rough table with μ = 0.4, and is joined by a light string over a smooth pulley (see diagram) to particle B (8 kg) on a smooth slope, α = 30°.
Released from rest, the string is cut when B has moved 3 m. Find the further distance A slides before stopping.

A: T − 0.4 × 4 × 9.8 = 4 a. B: 8 × 9.8 × $\frac{1}{2}$ − T = 8 a

Adding: 9.8(8 × $\frac{1}{2}$ − 0.4 × 4) = 12 a, so a = 1.96

T = 4(1.96 + 0.4 × 9.8) = 23.52

When cut: v^2 = 2 × 1.96 × 3 = 11.76. Then A decelerates at μg = 3.92

Further distance = $\frac{11.76}{2 \times 3.92}$ = 1.5

Answer: 1.5 m

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5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = 2t^3 - 18t$.
Find the coordinates of the points on the curve where the gradient is 3.

$dx/dt = 4t$, $dy/dt = 6t^2 - 18$, so $dy/dx = \frac{6t^2 - 18}{4t}$

Set equal to 3: $6t^2 - 12t - 18 = 0$
$t = -1$ or $t = 3$
Answer: (2, 16) and (18, 0)

6 Alternate turns and sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B take turns, A first, to score; the first to score wins. P(A scores) = $\frac{1}{3}$, P(B scores) = $\frac{1}{6}$ each attempt.
Find P(A wins (a) on attempt 1 or 2, (b) overall).

P(A wins at attempt 1) = $\frac{1}{3}$; to reach attempt 2 both miss first: $(\frac{2}{3})(\frac{5}{6}) = \frac{5}{9}$

(a) $\frac{1}{3} + \frac{5}{9} \times \frac{1}{3} = \frac{14}{27}$

(b) P(A wins) = $\frac{1}{3} + \frac{5}{9} \times \frac{1}{3} + \frac{5}{9}^2 \times \frac{1}{3} + ...$

Geometric: first term $\frac{1}{3}$, common ratio $\frac{5}{9}$

$S = \frac{a}{1 - r} = \frac{\frac{1}{3}}{\frac{4}{9}} = \frac{3}{4}$

Answer: (a) $\frac{14}{27}$ (b) $\frac{3}{4}$

7 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 5x^2 - 50x + 132$.

Complete the square: $y = 5(x - 5)^2 + 7$.
Step 1: $y = x^2 \rightarrow y = 5x^2$: a stretch, scale factor 5, parallel to the y-axis.
Step 2: replace x by $(x - 5)$ and add 7: translation by $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

Answer: a stretch, scale factor 5, parallel to the y-axis, then translation $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$

8 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = e^{3x}$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi\int_0^1 y^2\,dx = \pi\int_0^1 (e^{3x})^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (-\frac{1}{6} + \frac{1}{6}e^6)$

Answer: $\frac{1}{6}\pi(-1 + e^6)$

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9 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(2x + \frac{2}{x^2})^{12}$.

General term: $C(12,r)(2x)^{12-r}(\frac{2}{x^2})^r$.

Power of x: $1(12 - r) - 2r = 0$, so $r = 4$.

Term = $C(12,4) \times 2^8 \times (2)^4 = 2027520$.

Answer: 2027520

10 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-2x - 3}{(x + 2)^2}$ in the form $\frac{A}{x + 2} + \frac{B}{(x + 2)^2}$.

(b) Find the exact value of $\int_0^3 \frac{-2x - 3}{(x + 2)^2}\,dx$.

$-2x - 3 \equiv A(x + 2) + B$, so $A = -2$, and $B = 1$

Integral = $[-2\ln(x + 2) - \frac{1}{x + 2}]$

Substitute limits 0 and 3

Answer: (a) $A = -2$, $B = 1$; (b) $\frac{3}{10} + 2\ln 2 - 2\ln 5$