Day 295 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ab^x$ passes through (1, 18) and (5, 1458).
Find the exact values of a and b, and the value of y when x = 7.

$ab^1 = 18$ and $ab^5 = 1458$

Divide: $b^4 = 81$ so $b = 3$

$a = 18 \div 3^1 = 6$

$y = 6 \times 3^7$

Answer: a = 6, b = 3, y = 13122

2 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 11 m s^{-1} from the top of a cliff. The top of the cliff is 34.4 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-34.4 = 11t - 4.9t^2$, so $4.9t^2 - 11t - 34.4 = 0$

$$t = \frac{11 + \sqrt{(11^2 + 19.6 \times 34.4)}}{9.8} = \frac{11 + 28.2}{9.8} = 4$$

Height risen above the cliff top $= \frac{11^2}{(2 \times 9.8)} = 6.17$ m

Total distance $= 2 \times 6.17 + 34.4$

Answer: (a) 4 s (b) 46.7 m

3 Binomial: mean and variance given (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A binomial random variable X has mean 7 and variance 5.6.
(a) Find n and p.
(b) Find P(X = 6), correct to 3 significant figures.

$np = 7$ and $np(1 - p) = 5.6$

Divide: $1 - p = 5.6 \div 7 = \frac{4}{5}$, so $p = \frac{1}{5}$

$n = 7 \div \frac{1}{5} = 35$

$P(X = 6) = C(35, 6) (\frac{1}{5})^6 (\frac{4}{5})^{29} = 0.161$

Answer: (a) n = 35, p = $\frac{1}{5}$ (b) 0.161

4 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x + 2y = -8$
$2x^2 - xy - y^2 = -7$

From the first equation $x = -8 - 2y$.
Substitute into the second equation and collect terms:
this gives $9y^2 + 72y + 135 = 0$.
$y = -5$ or $y = -3$, giving $x = 2$ or $x = -2$.
Answer: (x, y) = (2, −5) and (−2, −3)

5 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 46 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 48.8 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 46$, and $T_2 = \frac{3}{4} T_1$

$T_1(\frac{4}{5} + \frac{3}{4} \times \frac{3}{5}) = 46$, so $T_1 = 36.8$ and $T_2 = 27.6$

The tension in AP is the larger: it is 36.8 when W = 46

Tensions are proportional to W, so $W_{max} = 46 \times \frac{48.8}{36.8} = 61$

Answer: (a) AP: 36.8 N, BP: 27.6 N (b) 61 N

6 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 3n - 4$, $u_1 = 3$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 730$.

$u_n = u_1 + \sum_{k=1}^{n-1}(3k - 4) = 3 + 3 \times \frac{(n-1)n}{2} - 4(n-1)$.

$= \frac{3}{2}n^2 - \frac{11}{2}n + 7$.

So $a = \frac{3}{2}$, $b = -\frac{11}{2}$, $c = 7$.

$u_{23} = 674 \leq 730$ and $u_{24} = 739 > 730$.

Answer: (a) $a = \frac{3}{2}$, $b = -\frac{11}{2}$, $c = 7$ (b) $n = 24$

7 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{1}{10 + 3\sin\theta + 4\cos\theta}$

(a) Write $3\sin\theta + 4\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{3^2 + 4^2} = 5$, $\tan\alpha = \frac{4}{3}$, $\alpha = 53.1°$

Denominator $= 10 + 5\sin(\theta + 53.1°)$, which lies between 5 and 15

Greatest $f(\theta) = \frac{1}{5} = \frac{1}{5}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{15} = \frac{1}{15}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 53.1° = 270°$, so $\theta = 216.9°$

Answer: (a) $5\sin(\theta + 53.1°)$ (b) greatest $\frac{1}{5}$, least $\frac{1}{15}$, $\theta = 216.9°$

8 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + 3y^2 + 2y$.

Find dy/dx and d^2y/dx^2 when $y = -2$.

$dx/dy = 3y^2 + 6y + 2 = 2$ when $y = -2$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{2}$

$d^2x/dy^2 = -6$ when $y = -2$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{-6}{2^3}$

Answer: $dy/dx = \frac{1}{2}$, $d^2y/dx^2 = \frac{3}{4}$

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9 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle with equation $(x + 0)^2 + (y + 0)^2 = 25$
passes through P(−3, 4). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (0, 0). Gradient of radius CP = $-\frac{4}{3}$.

Tangent gradient = $\frac{3}{4}$: $-3x + 4y = 25$.

x = 0: y = $\frac{25}{4}$. y = 0: x = $-\frac{25}{3}$.

Area = ½ × $\frac{25}{3}$ × $\frac{25}{4}$.

Answer: $\frac{625}{24}$

10 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A battery's life, in hours, is $N(57, 12^2)$.
Given that it exceeds 69, find the probability that it exceeds 78.

Required = $P(X > 78 \mid X > 69) = \frac{P(X > 78)}{P(X > 69)}$

$P(X > 78) = P(Z > 1.750) = 0.04006$

$P(X > 69) = P(Z > 1.000) = 0.1587$

$= \frac{0.04006}{0.1587} = 0.252$

Answer: 0.252