Day 293 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 5 strips to estimate

$\int_0^1 \frac{1}{\sqrt{1+x}}\,dx$, to 4 s.f.

Is it an overestimate or an underestimate? Explain.

h = $\frac{1}{5}$; ordinates at x = 0, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, …, 1

Sum = $\frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$ = 0.8295

f''(x) > 0 on the interval, so the curve is convex and the trapezia lie above the curve

Answer: 0.8295; overestimate

2 Coded data: mean, standard deviation, Σx² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 50 observations x, $\Sigma(x - 60) = 100$ and $\Sigma(x - 60)^2 = 1100$.

(a) Find the mean of x.

(b) Find the standard deviation of x.

(c) Find the value of Σx^2.

Mean of $(x - 60) = \frac{100}{50} = 2$, so mean of x = 62

Variance = $\frac{1100}{50} - (2)^2 = 18$

$\sigma = \sqrt{18}$

$\Sigma x^2 = \Sigma(x - 60)^2 + 120\,\Sigma(x - 60) + 50 \times 60^2$

= 1100 + 12000 + 180000

Answer: (a) 62 (b) 4.24 (c) 193100

3 Newton-Raphson iteration (AQA, Edexcel, OCR, MEI, Eduqas (Newton-Raphson))

$f(x) = e^x + 4x - 4$.

(a) Show that f(x) = 0 has a root α with $0 < \alpha < 1$.

(b) Use Newton-Raphson with $x_0 = 1$ to find x_1 and x_2 to 4 decimal places.

f(0) = −3.00 and f(1) = 2.72: a change of sign, f continuous

$f'(x) = e^x + 4$, so $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{2.718}{6.718} = 0.5954$

$x_2 = 0.5954 - \frac{0.1953}{5.814} = 0.5618$

Answer: x_1 = 0.5954, x_2 = 0.5618

4 Circle touching the axes (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle in the diagram touches both axes and passes through P. Find the equation of the circle, giving it in the form $x^2 + y^2 + ax + by + c = 0$.

Touches both axes in the first quadrant: centre (r, r), radius r.

P(5, 10) lies on the circle: $(5 - r)^2 + (10 - r)^2 = r^2$.

$r^2 - 30r + 125 = 0$, so r = 5 or r = 25.

The diagram shows P on the larger circle, so r = 25.

$(x - 25)^2 + (y - 25)^2 = 625$, then expand.

Answer: $x^2 + y^2 - 50x - 50y + 625 = 0$

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5 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a apple is N(93, 5^2) g. A apple is 'heavy' if it exceeds 101.
(a) Find the probability that a apple is heavy.
(b) 13 apples are chosen at random. Find the probability that at least 2 are heavy.

(a) P(X > 101) = P(Z > 1.600) = 0.05480
(b) Let Y = number heavy, Y ~ B(13, 0.05480)
P(Y ≥ 2) = 1 − P(Y ≤ 1) = 0.157
Answer: (a) 0.0548 (b) 0.157

6 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 450 kg, carrying a passenger of mass 80 kg, is supported by a vertical cable. The lift is moving upwards and accelerating at 0.4 m s^{-2}.
Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is upwards
Passenger: R − 80 × 9.8 = 80 × 0.4
R = 80 × (9.8 + 0.4) = 816
Lift and passenger: T − 530 × 9.8 = (450 + 80) × 0.4
T = 530 × 10.2 = 5406
Answer: (a) 816 N (b) 5406 N

7 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 3x + 2 > 0$ and $3x + 8 > 3$.

Quadratic: (x − 1)(x − 2) > 0 gives x < 1 or x > 2.

Linear: 3x + 8 > 3 gives $x > -\frac{5}{3}$.

Both must hold: take the overlap of the two sets.

Answer: $-\frac{5}{3} < x < 1$ or $x > 2$

8 Particle on a rough table, string slack when B lands (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (8 kg) and B (7 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a rough table with μ = 0.5, and B is 0.5 m above the floor.
The system is released from rest. Find the total distance A moves before coming to rest (A does not reach the pulley).

Until B lands, A: T − 0.5 × 8 × 9.8 = 8 a. B: 7 × 9.8 − T = 7 a
Adding: (7 − 0.5 × 8) × 9.8 = 15 a, so a = 1.96
$v^2 = 2 \times 1.96 \times 0.5 = 1.96$, so v = 1.4

After B lands, A decelerates at μg = 4.9: extra distance = $\frac{1.4^2}{2 \times 4.9} = 0.2$

Total distance = 0.5 + 0.2
Answer: 0.7 m

9 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $\frac{4x + 7}{(x + 3)(x + 2)}$ in partial fractions.

(b) Hence find the exact value of $\int_0^3 \frac{4x + 7}{(x + 3)(x + 2)}\,dx$.

Let N/D = $\frac{A}{x + 3} + \frac{B}{x + 2}$: 4x + 7 ≡ A(x + 2) + B(x + 3)
x = −3 gives A = 5; x = −2 gives B = −1
Integral = [5 ln(x + 3) − 1 ln(x + 2)]
Substitute limits 0 and 3
Answer: (a) 5/(x + 3) + (−1)/(x + 2); (b) 6ln 2 − ln 5

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10 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{2x}{x+1}$.

(a) Write down the equations of the asymptotes of C.
(b) The line $y = 2x + k$ is a tangent to C.
Find the possible values of k.

(a) Vertical: $x = -1$. As $x \to \infty$, $y \to 2$, so $y = 2$.
(b) $2x = (2x + k)(x + 1)$.
$2x^2 + kx + (k) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 8k = 0$.
Answer: (a) $x = -1$, $y = 2$ (b) $k = 0$ or $k = 8$