Day 291 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 14 and upper quartile 26. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 12 observations have mean 24. The largest is 47, an outlier, and it is removed. Find the new mean.

IQR = 26 − 14 = 12, so 1.5 × IQR = 18

Lower limit = 14 − 18 = −4, upper limit = 26 + 18 = 44

$\Sigma x = 12 \times 24 = 288$, so without 47: $\Sigma x = 241$

New mean = $\frac{241}{11}$

Answer: (a) −4 to 44 (b) 21.91

2 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 25 kg is at rest on rough horizontal ground ($\mu = 0.1$). A horizontal force of 59.5 N acts on it for 7 s and is then removed.

Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take $g = 9.8\ \text{m s}^{-2}$.

Friction = $\mu mg = 0.1 \times 25 \times 9.8 = 24.5$ N

While pushed: 59.5 − 24.5 = 25 a, so a = 1.4

$v = 1.4 \times 7 = 9.8$; $s_1 = \frac{1}{2} \times 1.4 \times 7^2 = 34.3$

After release: deceleration = $\mu g = 0.98$, so $s_2 = \frac{9.8^2}{2 \times 0.98} = 49$

Total distance = 34.3 + 49

Answer: (a) 9.8 m s^{-1} (b) 83.3 m

3 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$\log_2 x + \log_2 y = 4$

$x - y = 6$

$\log_2(xy) = 4$, so $xy = 2^4 = 16$

$x = y + 6$: $y^2 + 6y - 16 = 0$

$(y - 2)(y + 8) = 0$; $y > 0$ so $y = 2$

Then $x = y + 6$

Answer: x = 8, y = 2

4 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(x - \frac{3}{x^2})^9$.

General term: $C(9,r)(x)^{9-r}(-\frac{3}{x^2})^r$.

Power of x: $1(9 - r) - 2r = 0$, so $r = 3$.

Term = $C(9,3) \times 1^6 \times (-3)^3 = -2268$.

Answer: −2268

5 Without replacement: find the number of counters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 6 red counters and some white counters, n in total.

Two are taken without replacement. P(both red) = $\frac{5}{22}$. Find n.

P(both red) = $\frac{6}{n} \times \frac{5}{(n-1)} = \frac{30}{n(n-1)}$

So $n(n - 1) = 30 \div \frac{5}{22} = 132$

$n^2 - n - 132 = 0$, so $(n - 12)(n + 11) = 0$

$n > 0$, so $n = 12$

Answer: n = 12

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6 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each year Jo pays £1000 into an account on 1 January.
Interest of 2% is added each 31 December.
Find the first year-end at which the balance exceeds £8,000.

After n years: $1000(1.02) + 1000(1.02^2) + \ldots + 1000(1.02^n)$.

GP: $a = 1000 \times 1.02$, $r = 1.02$: $S_n = \dfrac{1000 \times 1.02\,(1.02^n - 1)}{0.02}$.

$S_n > 8000 \Rightarrow 1.02^n > 1.1569 \Rightarrow n > 7.358$.

So n = 8 years.

Answer: after 8 years

7 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -6 \\ 6 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

The point P has coordinates (−2, 7, −4).
Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathrm{FP} = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} - t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

FP · d = 0: 6 − t(6) = 0, so t = 1

F = (−4, 5, −2)

$\mathrm{PF} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$, so $\mathrm{PF} = \sqrt{12}$

Answer: F = (−4, 5, −2), PF = $2\sqrt{3}$

8 R-form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express 21 cos θ + 20 sin θ in the form R cos(θ − α), where R > 0 and 0 < α < 90°. Give α to 1 d.p.
(b) Hence solve 21 cos θ + 20 sin θ = −9 for 0 ≤ θ < 360°, giving answers to 1 d.p.

$R = \sqrt{21^2 + 20^2} = 29$ and $\tan\alpha = \dfrac{20}{21}$, so α = 43.6°

So 21 cos θ + 20 sin θ ≡ 29 cos(θ − 43.6°)

$\cos(\theta - \alpha) = \dfrac{-9}{29}$

θ − α = ±108.1° (using α = 43.6°)

Adding or subtracting multiples of 360° to give 0 ≤ θ < 360°

Answer: (a) R = 29, α = 43.6° (b) θ = 151.7°, 295.5°

9 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
x = 4 + 5cos t, y = −4 + 5sin t.
Find its Cartesian equation.

$\cos t = \dfrac{x - 4}{5}$, $\sin t = \dfrac{y + 4}{5}$.

Use $\cos^2 t + \sin^2 t = 1$:

$(x - 4)^2 + (y + 4)^2 = 25$.

Answer: $(x - 4)^2 + (y + 4)^2 = 25$

10 Just clearing a wall at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 45° to the horizontal. The ball just passes over the top of a vertical wall of height 40.8 m, at a horizontal distance of 80 m from O.

Find (a) the speed of projection, (b) the greatest height of the ball above the ground, to 3 s.f. Take g = 9.8 m s^{-2}.

At 45°: $x = \frac{u\,t}{\sqrt{2}}$ and $y = \frac{u\,t}{\sqrt{2}} - 4.9t^2$, so $y = x - \frac{9.8x^2}{u^2}$

$40.8 = 80 - \frac{9.8 \times 80^2}{u^2}$, so $u^2 = \frac{9.8 \times 6400}{39.2} = 1600$

$u = 40$

Greatest height $= \frac{(u \sin 45°)^2}{(2 \times 9.8)} = \frac{1600}{39.2}$

Answer: (a) 40 m s^{-1} (b) 40.8 m