Day 284	A Level Maths: S Tier 10-a-day
1 $f(x) = 3x^3 + ax^2 + bx - 36$, where a and b are constants. $(x + 3)$ is a factor of $f(x)$, and $f(x)$ leaves remainder 0 when divided by $(x + 2)$. Find a and b, and hence solve $f(x) = 0$.	
2 The fill volume of a bottle is $N(\mu, 9)$ with known variance 9. The mean was claimed to be 67 ml. A random sample of 49 has mean 68. Test at the 5% level whether the mean is less than 67.	
3 A box of weight 187 N is pulled at constant speed along rough horizontal ground ($\mu = \frac{8}{15}$) by a force P N at angle θ above the horizontal. (a) Find P when $\tan\theta = \frac{5}{12}$. (b) Find the least possible value of P as θ varies, and the corresponding θ to 1 d.p.	
4 A data set has lower quartile 35 and upper quartile 49. An outlier is a value more than 2 × IQR beyond the nearest quartile. (a) Find the range of values that are not outliers. (b) 12 observations have mean 49. The largest is 81, an outlier, and it is removed. Find the new mean.	
5 The equation $x^3 = 5x + 20$ has a root α. (a) Show that the equation can be written as $x = \sqrt[3]{(5x + 20)}$. (b) Use $x_{n+1} = \sqrt[3]{(5x_n + 20)}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p., and hence state α to 3 d.p.	

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6 Express $\frac{x^3 + 7x^2 + 16x + 16}{(x + 1)(x + 2)}$ in the form $ax + b + \frac{C}{(x + 1)} + \frac{D}{(x + 2)}$.	
7 Use integration by parts to find the exact value of $\int_0^1 x^2 e^{2x}\, dx$	
8 Solve $x^4 - 8x^3 + 15x^2 + 8x - 16 > 0$.	
9 Squares of side x cm are cut from the corners of a square sheet of side 12 cm and the sides folded up to make an open box. Find the value of x that maximises the volume, and that maximum volume.	
10 At time t = 0 hours, ship A is at the point with position vector $(18\mathbf{i} + 26\mathbf{j})$ km relative to a port O, and moves with constant velocity $(6\mathbf{i} - 5\mathbf{j})$ km h^{-1}. Ship B leaves O at t = 0 with constant speed 17 km h^{-1} and intercepts A. Find (a) the time at which B intercepts A, (b) the velocity of B as a vector.	