Day 282 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = 1$ and $y = -2$, so $p = -1$ and $q = -2$.

$y = \frac{a}{x - 1} - 2$.

P(3, 1): $1 = \frac{a}{2} - 2$, so $a = 6$.

Answer: a = 6, p = −1, q = −2

2 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

An arrow is projected from a point O on horizontal ground with speed 63.7 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 24.5^2 - 2 \times 9.8 \times H$, so $H = 30.625$

At height $\frac{H}{2} = 15.3125$: $v_y^2 = 24.5^2 - 2 \times 9.8 \times 15.3125 = 300.125$

$v_x = 58.8$ (constant), so speed$^2 = 3457.44 + 300.125 = 3757.565$

Answer: (a) 30.625 m (b) 61.3 m s^{-1}

3 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ab^x$ passes through (0, 6) and (4, 3750). Find the exact values of a and b, and the value of y when x = 5.

$ab^0 = 6$ and $ab^4 = 3750$

Divide: $b^4 = 625$ so $b = 5$

$a = 6 \div 5^0 = 6$

$y = 6 \times 5^5$

Answer: a = 6, b = 5, y = 18750

4 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 18 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin \alpha = \frac{4}{5}$ and μ = 0.55.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos \alpha = \frac{3}{5}$. Up the plane: $R = mg \cos \alpha$, so $a_{up} = g(\sin \alpha + \mu \cos \alpha) = 9.8(\frac{4}{5} + 0.55 \times \frac{3}{5}) = 11.074$

$0 = 18^2 - 2 \times 11.074 \times d$, so $d = \frac{324}{22.148} = 14.63$

Down the plane: $a_{down} = g(\sin \alpha - \mu \cos \alpha) = 9.8(\frac{4}{5} - 0.55 \times \frac{3}{5}) = 4.606$

$v^2 = 2 \times 4.606 \times 14.629 = 134.8$, so $v = 11.6$

Answer: (a) 14.6 m (b) 11.6 m s^{-1}

5 Expected winnings and a fair game (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 6 red, 3 blue and 2 green counters. A player takes two without replacement. Two reds win £2, two blues win £10, anything else wins nothing.
(a) Find the expected winnings.
(b) The player pays £1 to play. Find the expected profit.

P(RR) = $\frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

P(BB) = $\frac{3}{11} \times \frac{2}{10} = \frac{3}{55}$

(a) E = $2 \times \frac{3}{11} + 10 \times \frac{3}{55} = \frac{12}{11}$

(b) Expected profit = $\frac{12}{11} - 1 = \frac{1}{11}$

Answer: (a) £1.09 (b) £0.0909

6 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = \frac{x^2 + 7}{x + 4}$ at the point where x = 1
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At x = 1: y = $\frac{8}{5}$. Differentiate: gradient = $\frac{2}{25}$

Tangent: $y - \frac{8}{5} = \frac{2}{25}(x - 1)$, so $y = \frac{2}{25}x + \frac{38}{25}$

y-intercept $\frac{38}{25}$; x-intercept -19

Area = $\frac{1}{2} \times |\frac{38}{25}| \times |-19|$

Answer: $\frac{361}{25}$

7 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 2 and its common difference d is non-zero. The 1st, 2nd and 6th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: 2, 2 + d, 2 + 5d.
Geometric: $(2 + d)^2 = (2)(2 + 5d)$.
Expand: $d^2 + 4d + 4 = 10d + 4$, so $d(d - 6) = 0$.
$d \neq 0$, so d = 6. Ratio = $\frac{8}{2} = 4$.

Answer: d = 6, ratio = 4

8 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each year Jo pays £300 into an account on 1 January.
Interest of 3% is added each 31 December.
Find the first year-end at which the balance exceeds £15,000.

After n years: $300(1.03) + 300(1.03^2) + \ldots + 300(1.03^n)$.

GP: a = 300 × 1.03, r = 1.03: $S_n = \frac{300 \times 1.03\,(1.03^n - 1)}{0.03}$.

$S_n > 15000 \Rightarrow 1.03^n > 2.4563 \Rightarrow n > 30.402$.
So n = 31 years.
Answer: after 31 years

9 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{-x} \cos x$. Show that $dy/dx = e^{-x}(-\cos x - \sin x)$.
Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (-1)e^{-x} \cos x - e^{-x} \times \sin x$
$dy/dx = 0$ requires $-1 \cos x = 1 \sin x$
$\tan x = -1$, smallest positive $x = 2.3562$
$x = 2.36$
Second derivative test (or gradient change): a minimum
Answer: x = 2.36, minimum

10 Exponential and power models by logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data satisfy $y = ax^n$. The line of best fit for lg y against lg x is $\lg y = 1 + 1.5 \lg x$.
(a) Find a and n, giving a to 3 significant figures.
(b) Estimate y when x = 10, to 3 significant figures.

$y = ax^n$ gives $\lg y = \lg a + n \lg x$, so the gradient is n and the intercept is lg a
n = 1.5
$\lg a = 1$ so $a = 10^1 = 10.00$
$x = 10$: $y = 10.00 \times 10^{1.5}$
= 316
Answer: (a) a = 10.0, n = 1.5 (b) 316