Day 281 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(60, 0.25).
(a) State why a normal approximation is suitable for X.
(b) Use a normal approximation to estimate P(9 ≤ X ≤ 18), to 3 significant figures.

np = 15 and nq = 45, both greater than 5 (n large, p not extreme)
Y ~ N(15, 11.25) with σ = 3.354
Apply the continuity correction:
P(9 ≤ X ≤ 18) ≈ P(8.5 < Y < 18.5) = 0.8516 − 0.02632 = 0.825
Answer: 0.825

2 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 14 m s^{-1} from the top of a cliff. The top of the cliff is 92.4 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-92.4 = 14t - 4.9t^2$, so $4.9t^2 - 14t - 92.4 = 0$

$$t = \frac{14 + \sqrt{(14^2 + 19.6 \times 92.4)}}{9.8} = \frac{14 + 44.8}{9.8} = 6$$

Height risen above the cliff top = $\frac{14^2}{(2 \times 9.8)} = 10.0$ m

Total distance = 2 × 10.0 + 92.4
Answer: (a) 6 s (b) 112 m

3 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = -x^2 - 8x - 10$.

Complete the square: $y = -(x + 4)^2 + 6$.
Step 1: $y = x^2 \rightarrow y = -1x^2$: a reflection in the x-axis.
Step 2: replace x by (x + 4) and add 6: translation by $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$.
Answer: a reflection in the x-axis, then translation $\begin{pmatrix} \mathbf{-4} \\ \mathbf{6} \end{pmatrix}$

4 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\sec^2\theta - 5\tan\theta - 5 = 0$
for $0 \le \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $3\tan^2\theta - 5\tan\theta - 2 = 0$
$(\tan\theta - 2)(3\tan\theta + 1) = 0$
$\tan\theta = 2$ or $\tan\theta = -\frac{1}{3}$
θ = 63.4° and 161.6° (+180°)
Answer: θ = 63.4°, 161.6°, 243.4°, 341.6°

5 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - 13x^2 + 36 < 0$.

Use the factor theorem to find roots: −3, −2, 2, 3.
f(x) = (x + 3)(x + 2)(x − 2)(x − 3).
Sign of f in each region (left to right): +, −, +, −, +.
Take the regions where f < 0 (include roots if ≤ or ≥).
Answer: $-3 < x < -2$ or $2 < x < 3$

6 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\pi/2} (x + 2) \sin 2x \, dx$

Let u = x + 2 and dv/dx = sin 2x

Integral = $[-\frac{(x + 2)\cos 2x}{2}] + \int \frac{\cos 2x}{2} \, dx$

Answer: $2 + \frac{1}{4}\pi$

7 Grouped median with an unknown frequency (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the times, t minutes, taken by some runners.

Using linear interpolation, the median is estimated as 25 minutes. Find the value of k.

Total = 40 + k, so the median class is 20 to 30

Median = $20 + \frac{\frac{(40 + k)}{2} - (7 + k)}{13} \times 10 = 25$

Simplify: $13 - k/2 = 6.5$

k = 13

Answer: k = 13

8 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3x^3 - 11x^2 - 15x + 26}{(x - 4)(x - 1)}$ in the form $ax + b + \frac{C}{(x - 4)} + \frac{D}{(x - 1)}$.

Divide: the numerator has degree 3, the denominator degree 2.

$(3x^3 - 11x^2 - 15x + 26) \div (x^2 - 5x + 4)$ gives quotient 3x + 4.

Remainder: $-7x + 10$.

Put x = 4: remainder = C(3) so C = −6. Put x = 1: D = −1.

Answer: $3x + 4 - \frac{6}{(x - 4)} - \frac{1}{(x - 1)}$

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population N satisfies $\frac{dN}{dt} = -0.02N^2$, with N = 20 when t = 0.

Find N when t = 10, and the time taken for N to halve.

$\int N^{-2} \, dN = -0.02 \int dt$ gives $-\frac{1}{N} = -0.02t + c$

t = 0: $c = -\frac{1}{20}$, so $\frac{1}{N} = 0.02t + \frac{1}{20}$

t = 10: $\frac{1}{N} = \frac{1}{5} + \frac{1}{20}$; halving: $0.02t = \frac{1}{20}$

Answer: N = 4; t = $\frac{5}{2}$

10 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O travelling at a constant speed of 12 m s^{-1}. 9 s later, car B leaves O from rest with constant acceleration 2 m s^{-2}, travelling in the same direction.

B overtakes A at the point X. Find (a) the time B takes to reach X, (b) the distance OX, (c) the speed of B at X.

Let B travel for t s. A: distance = 12(t + 9); B: distance = ½ × 2 × t^2

Equate: $2t^2 - 24t - 216 = 0$, so $(t - 18)(2t + 12) = 0$

t = 18 (the other root is negative)

OX = $2 \times 18^2 / 2 = 324$ m, and speed = 2 × 18 = 36

Answer: (a) 18 s (b) 324 m (c) 36 m s^{-1}