Day 279 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−11, 13), B(−8, −8) and C(13, −5). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of AB: $3x - 21y = -81$.
Bisector of BC: $21x + 3y = 33$.
Solving: centre (1, 4).
$r^2 = (-11 - 1)^2 + (13 - 4)^2 = 225$.
Answer: $(x - 1)^2 + (y - 4)^2 = 225$

2 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 1 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 3 m s^{-2} to rest.
The journey takes 68 s and covers 870 m. Find V and the time spent at constant speed.

Time accelerating = V, time decelerating = $\frac{V}{3}$, so time at constant speed = $68 - V - \frac{V}{3}$

Area under the speed-time graph: $870 = 68V - \frac{V^2}{2} - \frac{V^2}{6}$

Multiply by 2ab: $4V^2 - 408V + 5220 = 0$
$V = 15$ or $V = 87$
$V = 87$ gives time at constant speed −48 s, which is negative, so $V = 15$
Time at constant speed = 68 − 15 − 5 = 48 s
Answer: V = 15, 48 s

3 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^3 - 3x^2 - 9x + 27 > 0$.

Use the factor theorem to find roots: −3, 3.
$f(x) = (x + 3)(x - 3)^2$.
Sign of f in each region (left to right): −, +, +.
Take the regions where $f > 0$ (include roots if ≤ or ≥).
Answer: $-3 < x < 3$ or $x > 3$

4 R-form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $12 \sin\theta - 9 \cos\theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.
(b) Hence solve $12 \sin\theta - 9 \cos\theta = 5$ for $0 \leq \theta < 360°$, giving answers to 1 d.p.

$R = \sqrt{12^2 + 9^2} = 15$ and $\tan\alpha = \frac{9}{12}$, so $\alpha = 36.9°$

So $12 \sin\theta - 9 \cos\theta \equiv 15 \sin(\theta - 36.9°)$

$\sin(\theta - \alpha) = \frac{5}{15}$

$\theta - \alpha = 19.5°$ or $160.5°$ (using $\alpha = 36.9°$)
Adding or subtracting multiples of 360° to give $0 \leq \theta < 360°$
Answer: (a) R = 15, α = 36.9° (b) θ = 56.3°, 197.4°

5 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1000 kg tows a trailer of mass 600 kg along a horizontal road, with driving force 1880 N. Resistances are 800 N on the car and 600 N on the trailer.

(a) Find the tension in the tow bar. At 8 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{1880 - 800 - 600}{1600} = 0.3$

Trailer: $T - 600 = 600 \times 0.3$, so $T = 780$

After the break the trailer decelerates at $\frac{600}{600} = 1$, and stops after $t = \frac{8}{1} = 8$ s

Trailer travels $\frac{8}{2} \times 8 = 32$ m

Car: $a = \frac{1880 - 800}{1000} = 1.08$, so $s = 8 \times 8 + ½ \times 1.08 \times 8^2 = 98.56$ m

Distance apart = $98.56 - 32$

Answer: (a) 780 N (b) 66.56 m

6 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $6\text{cosec}^2\theta - 7\cot\theta - 9 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $6\cot^2\theta - 7\cot\theta - 3 = 0$

$(2\cot\theta - 3)(3\cot\theta + 1) = 0$

$\cot\theta = \frac{3}{2}$ or $\cot\theta = -\frac{1}{3}$

$\tan\theta = \frac{2}{3}$ or -3, so $\theta = 33.7°$ and $108.4°$ $(+180°)$

Answer: θ = 33.7°, 108.4°, 213.7°, 288.4°

7 Normal: unknown standard deviation then a second probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times taken by a bus, in minutes are modelled by $N(70, \sigma^2)$. The probability of a value below 58 is 0.20.

(a) Find σ.

(b) Find the probability of a value below 55.

$P(X < 58) = 0.20$ so $Z = \frac{58 - 70}{\sigma} = -0.8416$

$\sigma = \frac{-12}{-0.8416} = 14.26$

$z = \frac{55 - 70}{14.26} = -1.052$

$P(X < 55) = P(Z < -1.052) = 0.146$

Answer: (a) 14.3 (b) 0.146

8 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(25 - 5x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(25 - 5x)^{\frac{1}{2}} = 25^{\frac{1}{2}}(1 - \frac{1}{5}x)^{\frac{1}{2}} = 5(1 + u)^{\frac{1}{2}}$, $u = -\frac{1}{5}x$.

$(1 + u)^{\frac{1}{2}} = 1 + \frac{1}{2}u + (-\frac{1}{8})u^2 + \frac{1}{16}u^3 + ...$

Substitute u and multiply by 5.

Valid when $|u| < 1$, i.e. $|x| < 5$.

Answer: $5 - \frac{1}{2}x - \frac{1}{40}x^2 - \frac{1}{400}x^3 + ...$; $|x| < 5$

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9 Poisson: sum of two independent variables, conditional (CIE)

X and Y are independent, X ~ Po(4) and Y ~ Po(3).
(a) State the distribution of X + Y.
(b) Given that X + Y = 3, find the probability that X = 1.

X + Y ~ Po(7)

$P(X = 1 \mid X + Y = 3) = \frac{P(X = 1)P(Y = 2)}{P(X + Y = 3)}$

$= \frac{e^{-4}4^1}{1! \times e^{-3}3^2/2!} \div \frac{e^{-7}7^3}{3!}$

= C(3, 1) $(\frac{4}{7})^1$ $(\frac{3}{7})^2$, a binomial probability

= 0.315

Answer: (a) Po(7) (b) 0.315

10 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}5\\5\\1\end{pmatrix} + t\begin{pmatrix}3\\0\\2\end{pmatrix}$.
The point P has coordinates (−3, 7, 0).
Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathrm{FP} = \begin{pmatrix}-8\\2\\-1\end{pmatrix} - t\begin{pmatrix}3\\0\\2\end{pmatrix}$

FP · d = 0: −26 − t(13) = 0, so t = −2

F = (−1, 5, −3)

$\mathrm{PF} = \begin{pmatrix}-2\\2\\3\end{pmatrix}$, so PF = $\sqrt{17}$

Answer: F = (−1, 5, −3), PF = $\sqrt{17}$