Day 278 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Poisson approximation to a binomial (CIE)

X ~ B(200, 0.015).
(a) Explain why a Poisson approximation is suitable and state its distribution.
(b) Use it to estimate P(X > 1), to 4 s.f.
(c) Find the exact value of P(X > 1), to 4 s.f.

n = 200 is large and p = 0.015 is small, with np = 3
X is approximately Po(3)
P(X > 1) ≈ 1 − P(Y ≤ 1) = 0.8009
Exact: 1 − P(X ≤ 1) with X ~ B(200, 0.015) = 0.8031
Answer: (a) Po(3) (b) 0.8009 (c) 0.8031

2 Geometric series: least n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 4.
Find the least n for which $S_n > 25651$.

$S_n = \frac{4(4^n - 1)}{3} > 25651$.

$4^n > 19239.25$.

$n > \frac{\log(1.924e+04)}{\log(4)} = 7.116$.

So the least n is 8.
Answer: n = 8

3 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 6 m and mass 50 kg rests on supports at C and D, where AC = 2 m and DB = 2 m. A girl of mass 40 kg stands on it at distance x m from A.
Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs R_C ≥ 0 and R_D ≥ 0. Limiting cases: R_C = 0 and R_D = 0 in turn

Tilting about D (girl beyond D): 50(1) = 40(x − 4), so $x = 4 + \frac{50 \times 1}{40} = 5.25$

Tilting about C (girl before C): 50(1) = 40(2 − x), so $x = 2 - \frac{50 \times 1}{40} = 0.75$

Answer: 0.75 ≤ x ≤ 5.25

4 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 3x^2 + 6x + 1$.

Complete the square: $y = 3(x + 1)^2 - 2$.
Step 1: $y = x^2 \rightarrow y = 3x^2$: a stretch, scale factor 3, parallel to the y-axis.
Step 2: replace x by (x + 1) and add −2: translation by $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.
Answer: a stretch, scale factor 3, parallel to the y-axis, then translation $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

5 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean air temperature (°C), Hurn, September 2015 (n = 184): mean 10.9, sd 3.7. (F = 1.8C + 32)
(a) Find the mean and sd in °F.
(b) Is a reading of 16.4 more than 2 sd above the mean?

New value = 1.8 × old + 32, so mean = 1.8 × 10.9 + 32 = 51.62
The constant shifts the mean but not the spread; only the multiplier affects σ
σ = 1.8 × 3.7 = 6.660
Outlier cut-off = mean + 2σ = 10.9 + 7.4 = 18.3 (original units)
16.4 is less than 18.3, so no
Answer: (a) mean 51.6, sd 6.66 (b) no

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6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = \ln t$, $y = 4t^2 + 2t$, $t > 0$.
Find the equation of the tangent where $t = 1$, in the form $y = mx + c$.

$dx/dt = \frac{1}{t}$, $dy/dt = 8t + 2$

$dy/dx = t(8t + 2)$ which is 10 at $t = 1$
At $t = 1$: $x = 0$, $y = 6$
Answer: $y = 10x + 6$

7 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(5r + 6) = 776$, find n.

$\sum_{r=1}^{n}(5r + 6) = 5 \times \frac{n(n + 1)}{2} + 6n.$

$\frac{5\,n(n + 1)}{2} + 6n = 776$, so $5n^2 + 17n - 1552 = 0$.

Solving $(n > 0)$: $n = 16$.
Answer: $n = 16$

8 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$.

The point P has coordinates $(1, -5, 4)$.
Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathrm{FP} = \begin{pmatrix} -5 \\ -3 \\ -3 \end{pmatrix} - t\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$\mathrm{FP} \cdot \mathbf{d} = 0$: $-16 - t(8) = 0$, so $t = -2$
F = $(2, -2, 3)$
$\mathrm{PF} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}$, so $\mathrm{PF} = \sqrt{11}$
Answer: F = $(2, -2, 3)$, PF = $\sqrt{11}$

9 Projectile through a given point: two angles (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 21 m s^{-1}, at angle θ above the horizontal. It passes through the point P, 27 m horizontally and 8 m vertically from O.
Find the two possible values of $\tan\theta$, and the two possible values of θ to 1 d.p. Take $g = 9.8$ m s^{-2}.

Horizontal: $x = 21\cos\theta \times t$. Vertical: $y = 21\sin\theta \times t - 4.9t^2$

Eliminating t: $y = x\tan\theta - \frac{9.8x^2}{2u^2}(1 + \tan^2\theta)$

$\frac{9.8 \times 27^2}{2 \times 21^2} = 8.1$, so $8 = 27\tan\theta - 8.1(1 + \tan^2\theta)$

$8.1\tan^2\theta - 27\tan\theta + 16.1 = 0$

$\tan\theta = \frac{7}{9}$ or $\frac{23}{9}$, so $\theta = 37.9°$ or $68.6°$

Answer: $\tan\theta = \frac{7}{9}$ or $\frac{23}{9}$; $\theta = 37.9°$ or $68.6°$

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10 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + 2xy + 2y^2 = 16$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + 2y}{2x + 4y}$

Horizontal tangent when $2x + 2y = 0$, so $y = -\frac{2x}{2}$

Substitute into the equation of C: $x^2 + (2xy) + (2y^2) = 16$
Solve for the remaining coordinate
Answer: $(4, -4)$ and $(-4, 4)$