Day 274 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 3^x - 4x$ has one stationary point.
Find its coordinates to 3 s.f. and show that it is a minimum.

$dy/dx = 3^x \ln 3 - 4 = 0$

$3^x = \frac{4}{\ln 3}$, $x = \frac{1}{\ln 3} \ln(\frac{4}{\ln 3}) = 1.18$

$d^2y/dx^2 = 3^x(\ln 3)^2 > 0$ for all x: minimum

$y = \frac{4}{\ln 3} - 4 \times x = -1.06$

Answer: (1.18, −1.06)

2 Normal: find mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses of apples, in grams are normally distributed. 35% are less than 70 and 20% are greater than 82.
Find the mean and standard deviation, each correct to 3 significant figures.

Let $X \sim N(\mu, \sigma^2)$. $P(X < 70) = 0.35$ gives $\frac{70 - \mu}{\sigma} = -0.3853$

$P(X > 82) = 0.20$ gives $\frac{82 - \mu}{\sigma} = 0.8416$

Subtract: $\frac{12}{\sigma} = 0.8416 - (-0.3853) = 1.2269$

$\sigma = 9.780$

$\mu = 70 - (-0.3853)(9.780) = 73.77$

Answer: $\mu = 73.8$, $\sigma = 9.78$

3 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \ln x = \ln(x + 20) + \ln 1$,
stating clearly why one root is rejected.

$\ln x^2 = \ln(1(x + 20))$, so $x^2 = 1x + 20$

$x^2 - 1x - 20 = 0$, $(x - 5)(x + 4) = 0$

$x = -4$ is rejected since $\ln x$ is undefined for $x < 0$

Answer: $x = 5$

4 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 29.4 m s^{-1} from the top of a vertical cliff of height 44.1 m, and lands on horizontal ground.
Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.

Vertically (up positive): $-44.1 = -4.9t^2$, so $t^2 = 9$

$t = 3$ s (the positive root)

Horizontal distance = $29.4 \times 3 = 88.2$ m

On landing: $v_x = 29.4$, $v_y = 0 - 9.8 \times 3 = -29.4$

speed = $\sqrt{(29.4^2 + 29.4^2)} = 41.6$, tan angle = $\frac{29.4}{29.4}$, angle = 45.0° below the horizontal

Answer: (a) 3 s, 88.2 m (b) 41.6 m s^{-1} at 45.0° below the horizontal

5 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{2x + 3}$ at the point where x = 3
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At x = 3: y = 9. Differentiate: gradient = 4

Tangent: $y - 9 = 4(x - 3)$, so $y = 4x - 3$

y-intercept −3; x-intercept $\frac{3}{4}$

Area = $\frac{1}{2} \times |-3| \times |\frac{3}{4}|$

Answer: $\frac{9}{8}$

6 Velocity with a square-root term (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds, where $t \geq 0$, its velocity is v m s^{-1}, where $v = 30\sqrt{t} - 5t$.

Find (a) the greatest speed of P, (b) the greatest displacement of P from its starting point.

$\frac{dv}{dt} = \frac{30}{2\sqrt{t}} - 5 = 0$ when $\sqrt{t} = 3$, so $t = 9$

$v = 30 \times 3 - 5 \times 9 = 45$

$v = 0$ when $\sqrt{t}(30 - 5\sqrt{t}) = 0$: $t = 36$ ($t = 0$ is the start)

$s = \int v\,dt = \frac{2}{3} \times 30\, t^{\frac{3}{2}} - \frac{5}{2}t^2$, so at $t = 36$: $s = 4320 - 3240$

Answer: (a) 45 m s^{-1} (b) 1080 m

7 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{21x^2 - 2x - 26}{(x + 3)(3x - 4)^2}$ in the form

$\frac{A}{(x + 3)} + \frac{B}{(3x - 4)} + \frac{C}{(3x - 4)^2}$.

$21x^2 - 2x - 26 \equiv A(3x - 4)^2 + B(x + 3)(3x - 4) + C(x + 3)$.

Put $x = -3$: $169 = A(-13)^2$, so $A = 1$.

Put $x = \frac{4}{3}$: $\frac{26}{3} = C(\frac{13}{3})$, so $C = 2$.

Compare x^2: $21 = 9A + 3B$, so $B = 4$.

Answer: A = 1, B = 4, C = 2

8 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x - 5| < |-x - 5|$.

Square both sides: $(4x - 5)^2 < (-x - 5)^2$.

$(5x)(3x - 10) < 0$.

Critical values $x = 0$ and $x = \frac{10}{3}$.

The quadratic has positive coefficient of x^2, so choose the inner region.

Answer: $0 < x < \frac{10}{3}$

9 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.

Given the second is a W, P(first was a W) = $\frac{4}{5}$. Find x.

P(W then W) = 0.4x, P(second W) = $0.4x + 0.2(1 - x)$

P(first W | second W) = $\frac{0.4x}{0.4x + 0.2(1 - x)} = \frac{4}{5}$

$0.4x = \frac{4}{5}(0.4x + 0.2 - 0.2x)$

Solve the linear equation for x: $x = \frac{2}{3}$

Answer: $x = \frac{2}{3}$

10 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle with equation $(x + 0)^2 + (y + 2)^2 = 169$
passes through P(−12, −7). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (0, −2). Gradient of radius CP = $\frac{5}{12}$.

Tangent gradient = $-\frac{12}{5}$: $-12x - 5y = 179$.

$x = 0$: $y = -\frac{179}{5}$. $y = 0$: $x = -\frac{179}{12}$.

Area = ½ × $\frac{179}{12}$ × $\frac{179}{5}$.

Answer: $\frac{32041}{120}$