Day 273 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has $u_3 = 5$ and $S_{10} = 125$.
(a) Find the first term and common difference.
(b) Find the least n for which $S_n > 844$.

u_3: $a + 2d = 5$. S_{10}: $10(2a + 9d)/2 = 125$, so $10a + 45d = 125$.
Solving: $a = -1$, $d = 3$.
$S_n = \frac{n}{2}(-2 + 3n - 3) = \frac{n(3n - 5)}{2}$.
Test: $S_{24} = 804 \leq 844$, $S_{25} = 875 > 844$.
Answer: (a) $a = -1$, $d = 3$ (b) $n = 25$

2 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos(2\theta + 10°) = \frac{\sqrt{2}}{2}$
for $-90° \leq \theta \leq 270°$.

Let $u = 2\theta + 10°$; then $\cos u = \frac{\sqrt{2}}{2}$
Base solutions: $u = 45°, -45°$ (+ multiples of 360°)
$\theta = (u - 10°) \div 2$, keeping those in range
Answer: $\theta = -27.5°, 17.5°, 152.5°, 197.5°$

3 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -3$ and $y = 1$, so $p = 3$ and $q = 1$.
$y = \frac{a}{x + 3} + 1$.
P$(-5, 4)$: $4 = \frac{a}{-2} + 1$, so $a = -6$.
Answer: $a = -6$, $p = 3$, $q = 1$

4 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 49 and 63.7 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take $g = 9.8$ m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 49
Moving upwards: $63.7 - 9.8t = 49$, so $t = 1.5$
Moving downwards: $63.7 - 9.8t = -49$, so $t = 11.5$
Horizontal distance $= 49 \times (11.5 - 1.5)$
Answer: (a) 1.5 s and 11.5 s (b) 490 m

5 Normal: unknown standard deviation then a second probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses of apples, in grams are modelled by $N(98, \sigma^2)$. The probability of a value below 88 is 0.20.
(a) Find σ.
(b) Find the probability of a value below 101.

$P(X < 88) = 0.20$ so $Z = \frac{88 - 98}{\sigma} = -0.8416$
$\sigma = \frac{-10}{-0.8416} = 11.88$
$z = \frac{101 - 98}{11.88} = 0.252$
$P(X < 101) = P(Z < 0.252) = 0.600$
Answer: (a) 11.9 (b) 0.600

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^3 + xy^2 = 10$.
Find the equation of the tangent to C at the point (2, 1),
in the form ax + by + c = 0 with integers a, b, c.

Differentiate implicitly with respect to x

At (2, 1): $dy/dx = -\frac{13}{4}$

$y - 1 = (-\frac{13}{4})(x - 2)$, then clear fractions

Answer: 13x + 4y − 30 = 0

7 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 20 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{4}{5}$ and μ = 0.4.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos\alpha = \frac{3}{5}$. Up the plane: R = mg cos α, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{4}{5} + 0.4 \times \frac{3}{5}) = 10.192$

$0 = 20^2 - 2 \times 10.192 \times d$, so $d = \frac{400}{20.384} = 19.62$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{4}{5} - 0.4 \times \frac{3}{5}) = 5.488$

$v^2 = 2 \times 5.488 \times 19.623 = 215.4$, so v = 14.7

Answer: (a) 19.6 m (b) 14.7 m s^{-1}

8 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 100 = 0$ are in the ratio
4 : 1. Find the possible values of k.

Let the roots be α and 4α.
Sum: 5α = −k. Product: $4\alpha^2 = 100$, so $\alpha^2 = 25$.
α = ±5, so k = −5α = ∓25.
Answer: k = ±25

9 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{-x}\sin 3x$. Show that $dy/dx = e^{-x}(-\sin 3x + 3\cos 3x)$.
Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (-1)e^{-x}\sin 3x + e^{-x} \times 3\cos 3x$
dy/dx = 0 requires $-1\sin 3x + 3\cos 3x = 0$
$\tan 3x = 3$, smallest positive 3x = 1.2490
x = 0.416
Second derivative test (or gradient change): a maximum
Answer: x = 0.416, maximum

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10 Distribution of the larger of two scores (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two fair 6-sided dice, numbered 1 to 6, are rolled. M is the smaller of the two scores.

(a) Show that $P(M = m) = \frac{(13 - 2m)}{36}$ for m = 1, 2, ..., 6.

(b) Find P(M = 4) and E(M).

(a) $M \geq m$ means both scores $\geq m$: $P(M \geq m) = (\frac{6 - m + 1}{6})^2$, so $P(M = m) = \frac{(6 - m + 1)^2 - (6 - m)^2}{36} = \frac{(13 - 2m)}{36}$

(b) $P(M = 4) = \frac{5}{36}$

$E(M) = \Sigma\, m\, P(M = m) = \frac{1}{36} \Sigma\, m(13 - 2m)$

$= \frac{91}{36}$

Answer: (b) $P(M = 4) = \frac{5}{36}$, $E(M) = \frac{91}{36}$