Day 271 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 recorded masses (grams) have mean 58 and standard deviation 2.
One value was recorded as 55 but should have been 53.
Find the correct mean and the correct standard deviation.

$\Sigma x = 20 \times 58 = 1160$, so corrected $\Sigma x = 1160 - 55 + 53 = 1158$

Correct mean $= \frac{1158}{20} = 57.90$

$\Sigma x^2 = 20(4 + 3364) = 67360$, corrected $\Sigma x^2 = 67360 - 3025 + 2809 = 67144$

Variance $= \frac{67144}{20} - 57.900^2 = 4.7900$, $\sigma = 2.189$

Answer: mean 57.9, standard deviation 2.19

2 Related rates of change (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ladder of length 25 m leans against a vertical wall. Its foot slides away from the wall at 3 m s^{-1}. When the foot is 7 m from the wall, find the rate at which the top is moving down.

$x^2 + y^2 = 625$, so $2x\,dx/dt + 2y\,dy/dt = 0$
When $x = 7$, $y = 24$

$dy/dt = -\frac{x}{y}\,dx/dt = -\frac{7}{24} \times 3$

Answer: $\frac{7}{8}$ m s^{-1}

3 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = -f(-2x - 8) + 1$.

Inside f: replace x by $-2x - 8$, so x-coordinates become $(x + 8) \div (-2)$.
Outside: y-coordinates become $-1 \times y + 1$.
A: $(-1, 0)$ B: $(-3, 4)$
C: $(-4, -2)$ D: $(-6, 4)$
Answer: A′(−1, 0), B′(−3, 4), C′(−4, −2), D′(−6, 4)

4 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A radioactive sample has mass $m = m_0 e^{-kt}$ grams, t in days.

Initially m = 200 and after 4 days $m = \frac{200}{3}$.

Find k, and the time for the mass to fall to 25 grams, to 3 s.f.

$m_0 = 200$. $200\,e^{-4k} = \frac{200}{3}$ so $e^{-4k} = \frac{1}{3}$

$k = \frac{1}{4} \ln 3 = 0.275$

$200\,e^{-kt} = 25$ so $e^{kt} = 8$, $t = \frac{1}{k} \ln 8$

$t = 7.57$ days
Answer: k = 0.275, t = 7.57 days

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5 Velocity parallel to a given direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves in a plane. At time t seconds its position vector relative to a fixed origin O is r metres, where $\mathbf{r} = (2t^2 - 9t)\mathbf{i} + (1.5t^2 - 8t)\mathbf{j}$.
Find (a) the value of t at which P is moving parallel to $(3\mathbf{i} + \mathbf{j})$, (b) the speed of P at this time (3 s.f. if not exact).

$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (4t - 9)\mathbf{i} + (3t - 8)\mathbf{j}$
Parallel to $(3\mathbf{i} + \mathbf{j})$: $(4t - 9) \times 1 = (3t - 8) \times 3$
$t(-5) = -15$, so $t = 3$
At $t = 3$: $\mathbf{v} = (3\mathbf{i} + \mathbf{j})$, speed $= \sqrt{(3^2 + 1^2)}$
Answer: (a) $t = 3$ (b) 3.16 m s^{-1}

6 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x\sqrt{108 - x}$, for $0 < x < 108$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{2(216 - 3x)}{2\sqrt{108 - x}}$
$\frac{dy}{dx} = 0$: $216 - 3x = 0$, so $x = 72$
$y = 2 \times 72 \times 6$
Gradient changes from positive to negative: a maximum
Answer: (72, 864), maximum

7 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -3$ and $y = 3$, so $p = 3$ and $q = 3$.
$y = \frac{a}{x + 3} + 3$.
P$(-4, 9)$: $9 = \frac{a}{-1} + 3$, so $a = -6$.
Answer: $a = -6$, $p = 3$, $q = 3$

8 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 1.5 per week. After a modification, 0 faults are found in 2 weeks. Test at the 1% level whether the rate of faults has decreased.

H_0: $\lambda = 1.5$ per week, H_1: $\lambda < 1.5$ per week
Over 2 weeks: $X \sim Po(3)$
$P(X \leq 0) = 0.0498$
$0.0498 > 1\%$, so do not reject H_0
There is insufficient evidence that the rate has decreased
Answer: Do not reject H_0 (p-value 0.0498)

9 Transformations: new equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 3x - 5$ is translated by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and then a stretch, scale factor 3, parallel to the y-axis.
Find the equation of the resulting curve in the form $y = ax^2 + bx + c$.

Translation by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$: $y = f(x - 1) + 1 = x^2 - 5x$.
Then a stretch, scale factor 3, parallel to the y-axis: multiply y by 3.
Expand and simplify to give the quadratic.
Answer: $y = 3x^2 - 15x$

10 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 6 m and mass 12 kg rests on supports at C and D, where AC = 2 m and AD = 4 m, with $R_C = 3R_D$.

Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.

$R_C = 3R_D$. Moments about A: $R_C \times 2 + R_D \times 4 = 12g \times AG$

$R_D(3 \times 2 + 4) = 12g \times AG$ with $R_D(1 + 3) = 12g$, so $AG = \frac{3 \times 2 + 4}{4} = 2.5$

Tilting about D: $R_C = 0$. Moments about D: $mg(6 - 4) = 12g(4 - 2.5)$

$m = \frac{12 \times 1.5}{2} = 9$

Answer: (a) 2.5 m (b) 9