Day 269 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{-1}{(x+1)} < \frac{-2}{(x-2)}$.

Multiply by the positive quantity $(x+1)^2(x-2)^2$:
$(-1)(x-(-1))(x-2)^2 < (-2)(x-2)(x-(-1))^2$.
Move to one side and factorise: $(x-(-1))(x-2)(x+4) < 0$.
Critical values: -1, 2, -4. Test the four regions.
Answer: $x < -4$ or $-1 < x < 2$

2 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sec(2\theta + 45°) = \sqrt{2}$
for $0° \leq \theta \leq 180°$.

Let $u = 2\theta + 45°$; then $\sec u = \sqrt{2}$
Base solutions: $u = 45$, $-45°$ (+ multiples of 360°)
$\theta = (u - 45°) \div 2$, keeping those in range
Answer: $\theta = 0°, 135°, 180°$

3 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 120 students in Year 10, 60 in Year 11 and 180 in the sixth form.
A stratified sample of 30 students is taken.
(a) Find the number sampled from each group.
(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $30 \times \frac{120}{360}$, $30 \times \frac{60}{360}$, $30 \times \frac{180}{360} = 10, 5, 15$
P(same) = $10 \times 9 + 5 \times 4 + 15 \times 14/30 \times 29$
$= \frac{320}{870} = 0.368$
Answer: (a) 10, 5, 15 (b) 0.368

4 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−7, 12) and B(−2, −13). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c-(-7))^2 + 144 = (c-(-2))^2 + 169$.
Expand and solve: $10c = -20$, so $c = -2$.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|(-5) \times (-13) - 0 \times 12| = \frac{65}{2}$.
Answer: C(−2, 0); area $\frac{65}{2}$

5 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 62 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.
Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 101.6 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$
Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 62$, and $T_2 = \frac{4}{3} T_1$
$T_1(\frac{3}{5} + \frac{4}{3} \times \frac{4}{5}) = 62$, so $T_1 = 37.2$ and $T_2 = 49.6$
The tension in BP is the larger: it is 49.6 when W = 62
Tensions are proportional to W, so $W_{max} = 62 \times \frac{101.6}{49.6} = 127$
Answer: (a) AP: 37.2 N, BP: 49.6 N (b) 127 N

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6 Self-inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{kx + 1}{x - 3}$ for $x \neq 3$. Given that f is self-inverse, find k, and hence solve $f(x) = x$.

Let $y = f(x)$: $y(x - 3) = kx + 1$, so $x(y - k) = 3y + 1$.

$f^{-1}(x) = \frac{3x + 1}{x - k}$.

$f = f^{-1}$ requires $k = 3$.

$f(x) = x$: $3x + 1 = x(x - 3)$, so $x^2 - 6x - 1 = 0$.

$x = 3 \pm \sqrt{10} = 3 \pm \sqrt{10}$.

Answer: $k = 3$; $x = 3 \pm \sqrt{10}$

7 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 10 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially $\theta = 95$, and after 10 minutes $\theta = 61$.
Find A and k, then find when $\theta = 48$ (3 s.f.).

$t = 0$: $10 + A = 95$ so $A = 85$

$t = 10$: $10 + 85e^{-10k} = 61$ so $e^{-10k} = \frac{3}{5}$

$k = \frac{1}{10} \ln \frac{5}{3} = 0.0511$

$85\, e^{-kt} = 38$ so $t = \frac{1}{k} \ln \frac{85}{38}$

$t = 15.8$ minutes

Answer: $A = 85$, $k = 0.0511$, $t = 15.8$ minutes

8 Pdf of the form a + bx with a given mean (CIE)

The pdf of X is $f(x) = a + bx$ for $0 \leq x \leq 4$, and 0 otherwise, where a and b are constants. $E(X) = \frac{11}{5}$.

(a) Find a and b.
(b) Find $P(X < 2)$.

Total area = 1: $4a + 8b = 1$

$E(X) = \int x(a + bx)\, dx = 8a + \frac{64}{3} b = \frac{11}{5}$

Solving the simultaneous equations: $a = \frac{7}{40}$, $b = \frac{3}{80}$

$P(X < 2) = \int f(x)\, dx$ from 0 to 2 $= \frac{7}{40} \times 2 + \frac{3}{80} \times 2$

$= \frac{17}{40}$

Answer: (a) $a = \frac{7}{40}$, $b = \frac{3}{80}$ (b) $\frac{17}{40}$

9 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from level ground with horizontal and vertical velocity components 34.3 and 44.1 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take $g = 9.8$ m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 34.3$
Moving upwards: $44.1 - 9.8t = 34.3$, so $t = 1$
Moving downwards: $44.1 - 9.8t = -34.3$, so $t = 8$
Horizontal distance $= 34.3 \times (8 - 1)$
Answer: (a) 1 s and 8 s (b) 240.1 m

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10 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{1}{8}) + \tan^{-1}(1))$.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{8} + 1}{1 - \frac{1}{8} \times 1} = \frac{\frac{9}{8}}{\frac{7}{8}}$$

Answer: $\mathbf{\frac{9}{7}}$