Day 264 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 463
inclusive that are divisible by 6 or by 10, but not by both.

Multiples of 6: 77 terms (6 to 462): sum = 18018.
Multiples of 10: 46 terms: sum = 10810.
Multiples of both, i.e. of 30: 15 terms: sum = 3600.
Required sum = 18018 + 10810 − 2 × 3600.
Answer: 21628

2 Screening test and Bayes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A faulty component is present in 2% of a population. A test is positive for 80% of those with it and for 5% of those without it.
Find P(positive) and P(has it | positive).

P(positive) = 0.02 × 0.8 + 0.98 × 0.05
= 0.016 + 0.049 = 0.065
P(has it | positive) = $\frac{0.016}{0.065}$
= 0.2462
Answer: (a) 0.065 (b) 0.246

3 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg, carrying a passenger of mass 90 kg, is supported by a vertical cable. The lift is moving downwards and accelerating at 0.4 m s^{-2}.
Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is downwards
Passenger: 90 × 9.8 − R = 90 × 0.4
R = 90 × (9.8 − 0.4) = 846
Lift and passenger: 490 × 9.8 − T = (400 + 90) × 0.4
T = 490 × 9.4 = 4606
Answer: (a) 846 N (b) 4606 N

4 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 + 10x + 24 > 0$ and $-2x - 6 > 1$.

Quadratic: (x + 6)(x + 4) > 0 gives x < −6 or x > −4.
Linear: $-2x - 6 > 1$ gives $x < -\frac{7}{2}$. (the inequality reverses as we divide by a negative)
Both must hold: take the overlap of the two sets.
Answer: $x < -6$ or $-4 < x < -\frac{7}{2}$

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5 Distribution of the larger of two scores (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two fair 10-sided dice, numbered 1 to 10, are rolled. M is the smaller of the two scores.

(a) Show that $P(M = m) = \frac{(21 - 2m)}{100}$ for m = 1, 2, ..., 10.

(b) Find P(M = 7) and E(M).

(a) M ≥ m means both scores ≥ m: $P(M \geq m) = (\frac{10 - m + 1}{10})^2$, so $P(M = m) = \frac{(10 - m + 1)^2 - (10 - m)^2}{100} = \frac{(21 - 2m)}{100}$

(b) $P(M = 7) = \frac{7}{100}$

$E(M) = \Sigma\, m\, P(M = m) = \frac{1}{100} \Sigma\, m(21 - 2m)$

$= \frac{77}{20}$

Answer: (b) $P(M = 7) = \frac{7}{100}$, $E(M) = \frac{77}{20}$

6 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 0.9 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 1.5 m s^{-2} to rest.

The journey takes 28 s and covers 180 m. Find V and the time spent at constant speed.

Time accelerating = $\frac{V}{0.9}$, time decelerating = $\frac{V}{1.5}$, so time at constant speed = $28 - \frac{V}{0.9} - \frac{V}{1.5}$

Area under the speed-time graph: $180 = 28V - \frac{V^2}{1.8} - \frac{V^2}{3}$

Multiply by 2ab: $2.4V^2 - 75.6V + 486 = 0$

V = 9 or V = 22.5

V = 22.5 gives time at constant speed −12 s, which is negative, so V = 9

Time at constant speed = 28 − 10 − 6 = 12 s

Answer: V = 9, 12 s

7 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the points of inflection of

$y = \frac{4}{x^2 + 27}$.

$dy/dx = -\frac{8x}{(x^2 + 27)^2}$

$d^2y/dx^2 = \frac{8(3x^2 - 27)}{(x^2 + 27)^3}$

Zero when $x^2 = 9$, so x = ±3, and the sign changes either side

$y = \frac{4}{36} = \frac{1}{9}$

Answer: $(3, \frac{1}{9})$ and $(-3, \frac{1}{9})$

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8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{3}{34 + 21\sin\theta + 20\cos\theta}$

(a) Write $21\sin\theta + 20\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{21^2 + 20^2} = 29$, $\tan\alpha = \frac{20}{21}$, $\alpha = 43.6°$

Denominator $= 34 + 29\sin(\theta + 43.6°)$, which lies between 5 and 63

Greatest $f(\theta) = \frac{3}{5} = \frac{3}{5}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{3}{63} = \frac{1}{21}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 43.6° = 270°$, so $\theta = 226.4°$

Answer: (a) $29\sin(\theta + 43.6°)$ (b) greatest $\frac{3}{5}$, least $\frac{1}{21}$, $\theta = 226.4°$

9 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 + 6x^3 + 8x^2 - 6x - 9 < 0$.

Use the factor theorem to find roots: -3, -1, 1.

$f(x) = (x + 3)^2(x + 1)(x - 1)$.

Sign of f in each region (left to right): +, +, −, +.

Take the regions where $f < 0$ (include roots if $\leq$ or $\geq$).

Answer: $-1 < x < 1$

10 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = 3t(6 - t)$,
for $0 \leq t \leq 6$. The curve meets the x-axis at O and at A.
Find the area enclosed between the curve and the x-axis.

$dx/dt = 4t$; x from 0 to 72 as t goes from 0 to 6

$\text{Area} = \int_0^6 y\,\frac{dx}{dt}\,dt = \int_0^6 12t^2(6 - t)\,dt$

$= 12[\frac{6t^3}{3} - \frac{t^4}{4}]$

Answer: 1296