Day 260 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Binomial: mean and variance given (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A binomial random variable X has mean 5 and variance 4.5.
(a) Find n and p.
(b) Find P(X = 6), correct to 3 significant figures.

$np = 5$ and $np(1 - p) = 4.5$

Divide: $1 - p = 4.5 \div 5 = \frac{9}{10}$, so $p = \frac{1}{10}$

$n = 5 \div \frac{1}{10} = 50$

$P(X = 6) = C(50, 6)\ (\frac{1}{10})^6\ (\frac{9}{10})^{44} = 0.154$

Answer: (a) n = 50, p = $\frac{1}{10}$ (b) 0.154

2 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population of bacteria is modelled by $P = P_0e^{kt}$, t in hours.
Initially P = 120 and after 5 hours P = 150.
Find k, and the time for the population to reach 1440, to 3 s.f.

$P_0 = 120$. $120\ e^{5k} = 150$ so $e^{5k} = \frac{5}{4}$

$k = \frac{1}{5} \ln \frac{5}{4} = 0.0446$

$120\ e^{kt} = 1440$ so $e^{kt} = 12$, $t = \frac{1}{k} \ln 12$

t = 55.7 hours

Answer: k = 0.0446, t = 55.7 hours

3 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A pile of logs has 15 logs in its bottom row. Each row above has 1 fewer logs than the row below, and the pile has as many rows as possible, each with at least one log. Find the number of rows and the total number of logs.

Row n has $15 - 1(n - 1) = -n + 16$ logs; need this ≥ 1.
$1(n - 1) \leq 14$, so $n \leq 15$: 15 rows.

Top row = 1 logs. $S = \frac{15}{2}(15 + 1) = 120$.

Answer: 15 rows, 120 logs

4 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 15 kg is at rest on rough horizontal ground ($\mu = 0.2$). A horizontal force of 50.4 N acts on it for 7 s and is then removed.

Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take $g = 9.8\ m\ s^{-2}$.

Friction = $\mu mg = 0.2 \times 15 \times 9.8 = 29.4$ N
While pushed: $50.4 - 29.4 = 15\ a$, so $a = 1.4$
$v = 1.4 \times 7 = 9.8$; $s_1 = ½ \times 1.4 \times 7^2 = 34.3$

After release: deceleration = $\mu g = 1.96$, so $s_2 = \frac{9.8^2}{2 \times 1.96} = 24.5$

Total distance = 34.3 + 24.5

Answer: (a) 9.8 m s^{-1} (b) 58.8 m

5 Regression line from summations (AQA, Edexcel, OCR, MEI)

For n = 12 pairs of values: $\Sigma x = 132$, $\Sigma y = 708$, $\Sigma x^2 = 1598$, $\Sigma xy = 7569$.
(a) Find the equation of the regression line of y on x.
(b) The x values lie between 7 and 16. Estimate y when x = 17 and comment on its reliability.

$S_{xx} = 1598 - \frac{132^2}{12} = 146$

$S_{xy} = 7569 - \frac{132 \times 708}{12} = -219$

$b = \frac{S_{xy}}{S_{xx}} = \frac{-219}{146} = -\frac{3}{2}$

$a = \bar{y} - b\,\bar{x} = 59 - (-\frac{3}{2})(11) = 75.5$

$y = 75.5 - 1.5x$

At x = 17: y = 50. This is extrapolation (outside the data range), so may be unreliable

Answer: (a) $y = 75.5 - 1.5x$ (b) 50, unreliable (extrapolation)

6 Triple angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (triple angle))

Given that $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$, solve
$\sin 3\theta = 2\sin\theta$ for $0 < \theta < 180°$.

Substitute: $\sin\theta(3 - 4\sin^2\theta - 2) = 0$

$\sin\theta = 0$ or $\sin^2\theta = \frac{1}{4}$

$\sin\theta = 0$ gives no solutions in the range

$\sin^2\theta = \frac{1}{4}$ gives 30°, 150°

Answer: θ = 30°, 150°

7 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{2x^3 + 3x^2 - 18x + 3}{(x-2)(x+3)}$ in the form $ax + b + \frac{C}{(x-2)} + \frac{D}{(x+3)}$.

Divide: the numerator has degree 3, the denominator degree 2.
$(2x^3 + 3x^2 - 18x + 3) \div (x^2 + x - 6)$ gives quotient $2x + 1$.
Remainder: $-7x + 9$.
Put x = 2: remainder = C(5) so C = −1. Put x = −3: D = −6.

Answer: $2x + 1 - \frac{1}{(x-2)} - \frac{6}{(x+3)}$

8 Velocity parallel to a given direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves in a plane. At time t seconds its position vector relative to a fixed origin O is r metres, where
$\mathbf{r} = (-2.5t^2 + 7t)\mathbf{i} + (1.5t^2 - 7t)\mathbf{j}$.
Find (a) the value of t at which P is moving parallel to $(2\mathbf{i} - \mathbf{j})$, (b) the speed of P at this time (3 s.f. if not exact).

$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (-5t + 7)\mathbf{i} + (3t - 7)\mathbf{j}$

Parallel to $(2\mathbf{i} - \mathbf{j})$: $(-5t + 7) \times -1 = (3t - 7) \times 2$
$t(-1) = -7$, so t = 7
At t = 7: $\mathbf{v} = (-28\mathbf{i} + 14\mathbf{j})$, speed $= \sqrt{(-28^2 + 14^2)}$

Answer: (a) t = 7 (b) 31.3 m s^{-1}

9 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ab^x$ passes through (0, 6) and (3, 750).
Find the exact values of a and b, and the value of y when x = 5.

$ab^0 = 6$ and $ab^3 = 750$

Divide: $b^3 = 125$ so $b = 5$

$a = 6 \div 5^0 = 6$

$y = 6 \times 5^5$

Answer: a = 6, b = 5, y = 18750

10 Tangent line to circle: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $8y = 6x + k$ is a tangent to the circle
$(x + 1)^2 + (y + 0)^2 = 4$.
Find the two possible values of k.

Substitute $y = \frac{6x + k}{8}$: $(x + 1)^2 + (\frac{6x + k}{8} + 0)^2 = 4$.

Multiply by 64 and collect: a quadratic in x with a = 100.

Tangent ⇒ discriminant = 0. This simplifies to $(k - 6)^2 = 400$.

$k - 6 = \pm 20$.

Answer: k = −14 or k = 26