Day 258	A Level Maths: S Tier 10-a-day
1 $f(x) = \frac{10}{2x^2 - 8x + 12}$ for all real x. Find the greatest value of f(x), the value of x at which it occurs, and the range of f.	
2 A car of mass 1000 kg tows a trailer of mass 300 kg along a horizontal road, with driving force 1665 N. Resistances are 400 N on the car and 225 N on the trailer. (a) Find the tension in the tow bar. At 6 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.	
3 Express $\frac{4x^2 - 20x + 29}{(x - 3)(x - 2)^2}$ in the form $\frac{A}{(x - 3)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$.	
4 A school has 180 students in Year 10, 60 in Year 11 and 240 in the sixth form. A stratified sample of 40 students is taken. (a) Find the number sampled from each group. (b) Two students are chosen at random from the sample. Find the probability they are from the same group.	
5 A is acute and B is acute. $\sin A = \frac{5}{13}$ and $\sin B = \frac{4}{5}$. Find the exact values of $\tan(A - B)$ and $\cos(A - B)$.	

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6 Given that $\sum_{r=1}^{n}(2r - 2) = 132$, find n.

7 Use the substitution $u = x^2 + 4$ to find the exact value of

$\int_0^1 x(x^2 + 4)^4 \, dx$

8 Two drones P and Q move so that, t seconds after launch, their position vectors in metres are

$\mathbf{r}_P = \begin{pmatrix} 4 \\ -4 \\ 5 \end{pmatrix} + t\begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} 4 \\ -11 \\ 1 \end{pmatrix} + t\begin{pmatrix} -6 \\ 1 \\ 6 \end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

9 A test of H_0: $p = 0.1$ against H_1: $p > 0.1$ uses a sample of 20 at the 1% level.
(a) Find the critical region for X, the number of successes.
(b) Find the actual significance level.
(c) Find P(Type II error) when in fact $p = 0.2$.

10 At time t = 0 hours, ship A is at the point with position vector $(12\mathbf{i} + 12\mathbf{j})$ km relative to a port O, and moves with constant velocity $(2\mathbf{i} + 4\mathbf{j})$ km h^{-1}. Ship B leaves O at t = 0 with constant speed 10 km h^{-1} and intercepts A.
Find (a) the time at which B intercepts A, (b) the velocity of B as a vector.