Day 256 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sin(3\theta - 45°) = \frac{1}{2}$
for $-180° \le \theta \le 180°$.

Let $u = 3\theta - 45°$; then $\sin u = \frac{1}{2}$
Base solutions: $u = 150°$, $30°$ (+ multiples of 360°)
$\theta = (u + 45°) \div 3$, keeping those in range
Answer: $\theta = -175°, -95°, -55°, 25°, 65°, 145°$

2 Binomial: mean and variance given (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A binomial random variable X has mean 21 and variance 8.4.
(a) Find n and p.
(b) Find P(X = 21), correct to 3 significant figures.

$np = 21$ and $np(1 - p) = 8.4$
Divide: $1 - p = 8.4 \div 21 = \frac{2}{5}$, so $p = \frac{3}{5}$
$n = 21 \div \frac{3}{5} = 35$
$P(X = 21) = C(35, 21) (\frac{3}{5})^{21} (\frac{2}{5})^{14} = 0.137$
Answer: (a) n = 35, p = $\frac{3}{5}$ (b) 0.137

3 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sec^2\theta - \tan\theta - 3 = 0$
for $0 \le \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $\tan^2\theta - \tan\theta - 2 = 0$
$(\tan\theta - 2)(\tan\theta + 1) = 0$
$\tan\theta = 2$ or $\tan\theta = -1$
$\theta = 63.4°$ and $135.0°$ (+180°)
Answer: $\theta = 63.4°, 135.0°, 243.4°, 315.0°$

4 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 39.2 m s^{-1} from the top of a vertical cliff of height 44.1 m, and lands on horizontal ground.
Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.

Vertically (up positive): $-44.1 = -4.9t^2$, so $t^2 = 9$
$t = 3$ s (the positive root)
Horizontal distance = $39.2 \times 3 = 117.6$ m
On landing: $v_x = 39.2$, $v_y = 0 - 9.8 \times 3 = -29.4$
speed = $\sqrt{(39.2^2 + 29.4^2)} = 49.0$, tan angle = $\frac{29.4}{39.2}$, angle = 36.9° below the horizontal
Answer: (a) 3 s, 117.6 m (b) 49.0 m s^{-1} at 36.9° below the horizontal

5 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 50 observations x, $\Sigma(x - 40) = 100$ and $\Sigma(x - 40)^2 = 1200$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 40) = \frac{100}{50} = 2$, so mean of $x = 42$

Variance $= \frac{1200}{50} - (2)^2 = 20$

$\sigma = \sqrt{20}$

$\Sigma x^2 = \Sigma(x - 40)^2 + 80\,\Sigma(x - 40) + 50 \times 40^2$

$= 1200 + 8000 + 80000$

Answer: (a) 42 (b) 4.47 (c) 89200

6 Velocity parallel to a given direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves in a plane. At time t seconds its position vector relative to a fixed origin O is r metres, where $\mathbf{r} = (0.5t^2 - 2t)\mathbf{i} + (t^2 - 9t)\mathbf{j}$.
Find (a) the value of t at which P is moving parallel to $(2\mathbf{i} - \mathbf{j})$, (b) the speed of P at this time (3 s.f. if not exact).

$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (t - 2)\mathbf{i} + (2t - 9)\mathbf{j}$

Parallel to $(2\mathbf{i} - \mathbf{j})$: $(t - 2) \times -1 = (2t - 9) \times 2$

$t(-5) = -20$, so $t = 4$

At $t = 4$: $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$, speed $= \sqrt{(2^2 + -1^2)}$

Answer: (a) $t = 4$ (b) 2.24 m s^{-1}

7 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x + 6| < -x + 6$.

Boundary values: $4x + 6 = -x + 6$ gives $x = 0$;
$4x + 6 = -(-x + 6)$ gives $x = -4$.

Also the corner of $|4x + 6|$ is at $x = -\frac{3}{2}$.

Testing each region (check the graph of each side) gives the solution set.

Answer: $-4 < x < 0$

8 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(8 - 2x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(8 - 2x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1 - \frac{1}{4}x)^{\frac{1}{3}} = 2(1 + u)^{\frac{1}{3}}$, $u = -\frac{1}{4}x$.

$(1 + u)^{\frac{1}{3}} = 1 + \frac{1}{3}u + (-\frac{1}{9})u^2 + \frac{5}{81}u^3 + \ldots$

Substitute u and multiply by 2.

Valid when $|u| < 1$, i.e. $|x| < 4$.

Answer: $2 - \frac{1}{6}x - \frac{1}{72}x^2 - \frac{5}{2592}x^3 + \ldots$; $|x| < 4$

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - xy + 3y^2 = 44$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -y}{-x + 6y}$

Horizontal tangent when $2x + -y = 0$, so $y = \frac{2x}{1}$

Substitute into the equation of C: $x^2 + (-xy) + (3y^2) = 44$
Solve for the remaining coordinate
Answer: $(-2, -4)$ and $(2, 4)$

10 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{-4x + 1}{x - 6} < -3$.

Rearrange: $\frac{-4x + 1}{x - 6} - (-3) < 0$.

Combine: $\frac{-x - 17}{x - 6} < 0$.

Critical values: $x = -17$ (zero of numerator), $x = 6$ (excluded, denominator zero).
Sign analysis of the three regions gives the solution.
Answer: $x < -17$ or $x > 6$