Day 254 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass M kg, carrying a passenger of mass 75 kg, hangs from a vertical cable. The tension in the cable is 5565 N and the floor exerts a force of 795 N on the passenger.

Find (a) the acceleration of the lift, with its direction, (b) the value of M. Take g = 9.8 m s^{-2}.

Passenger (upwards positive): $795 - 75 \times 9.8 = 75a$, so $a = 0.8$

$a = 0.8$ m s^{-2}, so the acceleration is upwards

Lift and passenger: $5565 - (M + 75) \times 9.8 = (M + 75) \times 0.8$

$(M + 75)(9.8 + 0.8) = 5565$, so $M + 75 = \frac{5565}{10.6} = 525$

Answer: (a) 0.8 m s^{-2} upwards (b) M = 450

2 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent.

Find x and y.

$P(A) = 3x + x = 4x$, $P(B) = x + y$, $P(A \cap B) = x$

Independent: $x = 4x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{4}$

Total probability: $3x + x + y + 0.15 = 1$, so $3x + (x + y) = 0.85$

$3x = 0.85 - \frac{1}{4} = 0.6$

$x = \frac{1}{5}$, $y = \frac{1}{4} - x = \frac{1}{20}$

Answer: $x = \frac{1}{5}$, $y = \frac{1}{20}$

3 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(-3\mathbf{i} + \mathbf{j})$ m with velocity $(-4\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At t = 8, P has velocity $(12\mathbf{i} + 21\mathbf{j})$ m s^{-1}.

Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{j})$, (c) the position vector of P at that time.

$a = \frac{v - u}{t} = \frac{(12\mathbf{i} + 21\mathbf{j}) - (-4\mathbf{i} - 3\mathbf{j})}{8} = (2\mathbf{i} + 3\mathbf{j})$

$v = (-4\mathbf{i} - 3\mathbf{j}) + t(2\mathbf{i} + 3\mathbf{j})$

Parallel to $(\mathbf{j})$: $(2t - 4) \times 1 = (3t - 3) \times 0$, giving $t = 2$

$r = (-3\mathbf{i} + \mathbf{j}) + 2(-4\mathbf{i} - 3\mathbf{j}) + \frac{1}{2} \times 2^2 (2\mathbf{i} + 3\mathbf{j}) = (-7\mathbf{i} + \mathbf{j})$

Answer: (a) $(2\mathbf{i} + 3\mathbf{j})$ m s^{-2} (b) t = 2 (c) $(-7\mathbf{i} + \mathbf{j})$ m

4 Fixed point iteration and rearrangement (AQA, Edexcel, OCR, MEI, Eduqas, CIE (iterative methods))

The equation $x^3 = 2x + 4$ has a root α.

(a) Show that the equation can be written as $x = \sqrt[3]{(2x + 4)}$.

(b) Use $x_{n+1} = \sqrt[3]{(2x_n + 4)}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p., and hence state α to 3 d.p.

Take the cube root of both sides to obtain $x = \sqrt[3]{(2x + 4)}$

$x_1 = 2.1544$, $x_2 = 2.0254$, $x_3 = 2.0042$

The iteration converges (successive values agree to 3 d.p.): $\alpha = 2.000$

Answer: $x_1 = 2.1544$, $x_2 = 2.0254$, $x_3 = 2.0042$; $\alpha = 2.000$

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5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-4x - 3| < 2x + 6$.

Boundary values: $-4x - 3 = 2x + 6$ gives $x = -\frac{3}{2}$;

$-4x - 3 = -(2x + 6)$ gives $x = \frac{3}{2}$.

Also the corner of $|-4x - 3|$ is at $x = -\frac{3}{4}$.

Testing each region (check the graph of each side) gives the solution set.

Answer: $\mathbf{-\frac{3}{2} < x < \frac{3}{2}}$

6 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary points of
$y = 2x + 4\cos x$ for $0 \leq x \leq \pi$,
and determine their nature.

$dy/dx = 2 - 4\sin x = 0$, so $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

$d^2y/dx^2 = -4\cos x$

Its sign at each point decides maximum or minimum

Answer: $\mathbf{(\frac{\pi}{6}, \frac{1}{3}\pi + 2\sqrt{3})}$ **maximum;** $\mathbf{(\frac{5\pi}{6}, \frac{5}{3}\pi - 2\sqrt{3})}$ **minimum**

7 Cubic with only one real root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - x^2 - 4$.

Show that $(x - 2)$ is a factor of $f(x)$, and that the equation $f(x) = 0$ has exactly one real root.

$f(2) = 8 - 4 - 4 = 0$, so $(x - 2)$ is a factor.

$f(x) = (x - 2)(x^2 + x + 2)$.

For $x^2 + x + 2 = 0$: $b^2 - 4ac = 1 - 8 = -7 < 0$.

No real roots from the quadratic, so the only real root is $x = 2$.

Answer: Proved; the only real root is x = 2

8 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bulb's lifetime, in hours, is $N(86, 5^2)$.

Given that it exceeds 91, find the probability that it exceeds 99.

$\text{Required} = P(X > 99 \mid X > 91) = \frac{P(X > 99)}{P(X > 91)}$

$P(X > 99) = P(Z > 2.600) = 0.004661$

$P(X > 91) = P(Z > 1.000) = 0.1587$

$= \frac{0.004661}{0.1587} = 0.0294$

Answer: 0.0294

9 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{2}{(x + 1)} < \frac{3}{(x - 4)}$.

Multiply by the positive quantity $(x + 1)^2(x - 4)^2$:

$2(x - (-1))(x - 4)^2 < 3(x - 4)(x - (-1))^2$.

Move to one side and factorise: $(x - (-1))(x - 4)(-x - 11) < 0$.

Critical values: -1, 4, -11. Test the four regions.

Answer: $\mathbf{-11 < x < -1}$ **or** $\mathbf{x > 4}$

10 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two drones P and Q move so that, t seconds after launch, their position vectors in metres are

$\mathbf{r}_P = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} 4 \\ -7 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$

$-5 + t(1) = 0$, so $t = 5$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, with length $\sqrt{1}$

Answer: t = 5, minimum distance = 1 m