Day 248 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $12\text{cosec}^2\theta + 7\cot\theta - 11 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $12\cot^2\theta + 7\cot\theta + 1 = 0$
$(3\cot\theta + 1)(4\cot\theta + 1) = 0$
$\cot\theta = -\frac{1}{3}$ or $\cot\theta = -\frac{1}{4}$
$\tan\theta = -3$ or -4, so $\theta = 108.4°$ and $104.0°$ $(+180°)$
Answer: θ = 104.0°, 108.4°, 284.0°, 288.4°

2 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 8 m and mass 60 kg rests on supports at C and D, where AC = 3 m and DB = 3 m. A girl of mass 50 kg stands on it at distance x m from A.
Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs $R_C \geq 0$ and $R_D \geq 0$. Limiting cases: $R_C = 0$ and $R_D = 0$ in turn
Tilting about D (girl beyond D): $60(1) = 50(x - 5)$, so $x = 5 + \frac{60 \times 1}{50} = 6.2$
Tilting about C (girl before C): $60(1) = 50(3 - x)$, so $x = 3 - \frac{60 \times 1}{50} = 1.8$
Answer: $1.8 \leq x \leq 6.2$

3 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact gradient of the curve $y = 6^{x-3}$
at the point where x = 7.

$y = e^{(x-3)\ln 6}$
$dy/dx = 1 \ln 6 \times 6^{x-3}$
At x = 7: exponent = 4, so $dy/dx = 1 \ln 6 \times 6^4$
Answer: 1296ln 6

4 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 11\sqrt{x} + 30 = 0$.

Let $u = \sqrt{x}$: $u^2 - 11u + 30 = 0$.
$(u - 5)(u - 6) = 0$, so $\sqrt{x} = 5$ or $\sqrt{x} = 6$.
Square: x = 25 or x = 36.
Answer: x = 25 or x = 36

5 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ab^x$. The graph of lg y against x is the straight line shown.
Find a and b to 3 significant figures.

$\lg y = \lg a + x \lg b$. The intercept is $\lg a = 0.5$, so $a = 10^{0.5} = 3.162$
Gradient $= \frac{1.4 - 0.5}{9} = 0.1$
$\lg b = 0.1$ so $b = 10^{0.1} = 1.259$
Answer: a = 3.16, b = 1.26

6 Inverse function and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The function f is defined by $f(x) = 3x^2 - 18x + 35$ for $x \geq k$. Given that k is the least value for which f is one-one, find k, $f^{-1}(x)$, and the domain of f^{-1}.

Complete the square: $f(x) = 3(x - 3)^2 + 8$.
The vertex is at x = 3, so k = 3 (f is one-one on one side of it).

Let y = f(x): $y - 8 = 3(x - 3)^2$, so $(x - 3) = \pm\sqrt{\frac{y - 8}{3}}$.

As $x \geq 3$, take the positive root: $x = 3 + \sqrt{\frac{y - 8}{3}}$.

Range of f = domain of f^{-1}: f(3) = 8, so $x \geq 8$.

Answer: $k = 3$; $f^{-1}(x) = 3 + \sqrt{\frac{x - 8}{3}}$; $x \geq 8$

7 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the stationary points of $y = 5\cos 2x + 10\cos x$ for $0 < x < 2\pi$, and determine their nature.

$dy/dx = -10\sin 2x - 10\sin x = -10\sin x(2\cos x + 1)$

$dy/dx = 0$: $\sin x = 0$ or $\cos x = -\frac{1}{2}$

$x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ in the range

Test the sign of d^2y/dx^2 at each

Answer: $(\frac{2\pi}{3}, -\frac{15}{2})$ minimum; $(\pi, -5)$ maximum; $(\frac{4\pi}{3}, -\frac{15}{2})$ minimum

8 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 observations x, $\Sigma(x - 40) = 60$ and $\Sigma(x - 40)^2 = 1290$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 40) = \frac{60}{40} = 1.5$, so mean of x = 41.5

Variance = $\frac{1290}{40} - (1.5)^2 = 30$

$\sigma = \sqrt{30}$

$\Sigma x^2 = \Sigma(x - 40)^2 + 80\,\Sigma(x - 40) + 40 \times 40^2$
$= 1290 + 4800 + 64000$

Answer: (a) 41.5 (b) 5.48 (c) 70090

9 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (2 m, 120 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{4}{3}$, where α is the angle between string and rod. A 20 N particle hangs at P, AP = 1 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. Moments about A: $T\sin\alpha \times 2 = 120 \times 1 + 20 \times 1$

$T \times \frac{4}{5} \times 2 = 140$, so T = 87.5

Horizontal: $H = T\cos\alpha = 52.5$. Vertical: $V + T\sin\alpha = 120 + 20$, so V = 70

Force = $\sqrt{(52.5^2 + 70^2)} = 87.5$, angle = $\tan^{-1}(\frac{70}{52.5}) = 53.1°$ above the horizontal

Answer: (a) 87.5 N (b) 87.5 N at 53.1° above the horizontal (away from the wall)

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10 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 219
inclusive that are divisible by 6 or by 10, but not by both.

Multiples of 6: 36 terms (6 to 216): sum = 3996.
Multiples of 10: 21 terms: sum = 2310.
Multiples of both, i.e. of 30: 7 terms: sum = 840.
Required sum = 3996 + 2310 − 2 × 840.
Answer: 4626