Day 243 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A tennis ball is projected from a point O on horizontal ground with speed 63.7 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{12}{5}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 24.5, vertical 58.8 (as $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$)

Time of flight: $0 = 58.8\,t - 4.9t^2$, so $t = \frac{58.8}{4.9} = 12$

Range = $24.5 \times 12 = 294$

Greatest height: $0 = 58.8^2 - 2 \times 9.8 \times H$, so $H = \frac{3457.44}{19.6} = 176.4$

Answer: (a) 12 s (b) 294 m (c) 176.4 m

2 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 4 per hour. After a modification, 19 faults are found in 2 hours. Test at the 10% level whether the rate of faults has increased.

H_0: $\lambda = 4$ per hour, H_1: $\lambda > 4$ per hour

Over 2 hours: X ~ Po(8)

$P(X \geq 19) = 1 - P(X \leq 18) = 0.000650$

$0.000650 < 10\%$, so reject H_0

There is sufficient evidence that the rate has increased

Answer: Reject H_0 (p-value 0.000650)

3 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 16 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{4}{5}$ and $\mu = 0.8$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos\alpha = \frac{3}{5}$. Up the plane: $R = mg\cos\alpha$, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{4}{5} + 0.8 \times \frac{3}{5}) = 12.544$

$0 = 16^2 - 2 \times 12.544 \times d$, so $d = \frac{256}{25.088} = 10.20$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{4}{5} - 0.8 \times \frac{3}{5}) = 3.136$

$v^2 = 2 \times 3.136 \times d = 16^2 \times \frac{3.136}{12.544} = 8^2$

Answer: (a) 10.2 m (b) 8 m s^{-1}

4 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{2}{9}) + \tan^{-1}(\frac{1}{2}))$.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{2}{9} + \frac{1}{2}}{1 - \frac{2}{9} \times \frac{1}{2}} = \frac{\frac{13}{18}}{\frac{8}{9}}$$

Answer: $\frac{13}{16}$

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5 Binomial approximation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(16 + x)^{\frac{1}{2}}$ up to and including the term in x^2.
(b) Use x = 0.03 to estimate $\sqrt{16.03}$ to 5 decimal places.

(a) $(16 + x)^{\frac{1}{2}} = 4(1 + \frac{1}{16}x)^{\frac{1}{2}}$.

$= 4 + \frac{1}{8}x - \frac{1}{512}x^2 + ...$

(b) 16 + x with x = 0.03 gives 16.03.

Substitute: $4 + \frac{1}{8}(0.03) + (-\frac{1}{512})(0.03)^2$.

= 4.00375 (5 d.p.).

Answer: (a) $4 + \frac{1}{8}x - \frac{1}{512}x^2$ (b) 4.00375

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations x = 4 cos t, y = 4 sin t.
Find a cartesian equation of the curve, with no fractions.

$\cos t = \frac{x}{4}$, $\sin t = \frac{y}{4}$

Use $\cos^2 t + \sin^2 t = 1$: $\frac{x^2}{16} + \frac{y^2}{16} = 1$

Multiply by 256

Answer: $16x^2 + 16y^2 = 256$

7 Poisson: sum of two independent variables, conditional (CIE)

X and Y are independent, X ~ Po(2) and Y ~ Po(2).
(a) State the distribution of X + Y.
(b) Given that X + Y = 4, find the probability that X = 1.

X + Y ~ Po(4)

$P(X = 1 \mid X + Y = 4) = \frac{P(X = 1)P(Y = 3)}{P(X + Y = 4)}$

$= \frac{e^{-2}2^1}{1! \times e^{-2}2^3/3!} \div \frac{e^{-4}4^4}{4!}$

$= C(4, 1)\ (\frac{1}{2})^1\ (\frac{1}{2})^3$, a binomial probability

= 0.250

Answer: (a) Po(4) (b) 0.250

8 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 407
inclusive that are divisible by 5 or by 7, but not by both.

Multiples of 5: 81 terms (5 to 405): sum = 16605.
Multiples of 7: 58 terms: sum = 11977.
Multiples of both, i.e. of 35: 11 terms: sum = 2310.
Required sum = 16605 + 11977 − 2 × 2310.

Answer: 23962

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - xy + 2y^2 = 28$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -y}{-x + 4y}$

Horizontal tangent when $2x + -y = 0$, so $y = \frac{2x}{1}$

Substitute into the equation of C: $x^2 + (-xy) + (2y^2) = 28$
Solve for the remaining coordinate
Answer: $(-2, -4)$ and $(2, 4)$

10 Binomial expansion with partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{4x - 4}{(1 + 2x)(1 - 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) in ascending powers of x up to x^3, stating the range of validity.

(a) $4x - 4 \equiv A(1 - 2x) + B(1 + 2x)$: $A = -3$, $B = -1$.

$f(x) = \frac{-3}{1 + 2x} - \frac{1}{1 - 2x}$.

(b) $\frac{1}{1 + cx} = 1 - cx + c^2x^2 - c^3x^3 + \ldots$ (use $c = 2$, then $c = -2$).

Coefficients of x^j: $-3(-2)^j - 1(2)^j$.

Valid for $|x| < \frac{1}{2}$ (both expansions).

Answer: (a) $A = -3$, $B = -1$ (b) $-4 + 4x - 16x^2 + 16x^3 + \ldots$; $|x| < \frac{1}{2}$