Day 242	A Level Maths: S Tier 10-a-day
1 $f(x) = 2x + 2$ and $g(x) = x^2 + 2$. Solve $fg(x) = gf(x)$.	
2 Class S has 20 students with mean mark 41 and standard deviation 3. Class R has 15 students with mean mark 78 and standard deviation 7. (a) Find the mean mark of all 35 students. (b) Find the standard deviation of all 35 students.	
3 $f(x) = x^3 + 4x^2 + 11x + 8$. Show that $(x + 1)$ is a factor of $f(x)$, and that the equation $f(x) = 0$ has exactly one real root.	
4 A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where $v = 2t^2 - 16t + 30$ for $0 \leq t \leq 6$ Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 6 seconds.	
5 A is acute and B is acute. $\cos A = \frac{5}{13}$ and $\sin B = \frac{21}{29}$. Find the exact values of $\sin(A + B)$ and $\cos(A + B)$.	

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6 The line $5y = 12x + k$ is a tangent to the circle $(x - 2)^2 + (y - 4)^2 = 1$. Find the two possible values of k.	
7 The equation $x^3 = 2x + 6$ has a root α. (a) Show that the equation can be written as $x = \sqrt[3]{(2x + 6)}$. (b) Use $x_{n+1} = \sqrt[3]{(2x_n + 6)}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p., and hence state α to 3 d.p.	
8 Two drones P and Q move so that, t seconds after launch, their position vectors in metres are $\mathbf{r}_P = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ -3 \\ 0 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} -2 \\ 10 \\ -5 \end{pmatrix} + t\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$. Find the time at which they are closest and the exact minimum distance between them.	
9 A box holds 8 components, each faulty with probability 0.05, independently. A box is rejected if it has at least 2 faulty components. (a) Find the probability that a box is rejected. (b) 11 boxes are inspected. Find the probability that at least 3 are rejected.	
10 A non-uniform rod AB of length 5 m and mass 6 kg rests on supports at C and D, where AC = 1 m and AD = 3 m, with R_C = $\frac{1}{3}$ R_D. Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.	