Day 238 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - 13x^2 + 36 \geq 0$.

Use the factor theorem to find roots: $-3, -2, 2, 3$.
$f(x) = (x + 3)(x + 2)(x - 2)(x - 3)$.
Sign of f in each region (left to right): $+, -, +, -, +$.
Take the regions where $f \geq 0$ (include roots if $\leq$ or $\geq$).
Answer: $x \leq -3$ or $-2 \leq x \leq 2$ or $x \geq 3$

2 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $e^x + 4e^{-x} = 5$,
giving your answers exactly in terms of natural logarithms.

Multiply by e^x: $e^{2x} - 5e^x + 4 = 0$
$(e^x - 1)(e^x - 4) = 0$
$e^x = 1$ or $e^x = 4$
Answer: $x = 0$ and $x = \ln 4$

3 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 115 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 96 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 115$, and $T_2 = \frac{4}{3} T_1$

$T_1(\frac{3}{5} + \frac{4}{3} \times \frac{4}{5}) = 115$, so $T_1 = 69$ and $T_2 = 92$

The tension in BP is the larger: it is 92 when W = 115

Tensions are proportional to W, so $W_{max} = 115 \times \frac{96}{92} = 120$

Answer: (a) AP: 69 N, BP: 92 N (b) 120 N

4 Proof by contradiction (irrationality) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove by contradiction that $\sqrt{19}$ is irrational.

Assume $\sqrt{19} = \frac{a}{b}$ with a, b integers and no common factor.

Then $19b^2 = a^2$, so a^2 is a multiple of 19.
As 19 is prime, a is a multiple of 19: let $a = 19k$.
Then $19b^2 = 361k^2$, so $b^2 = 19k^2$, and b is a multiple of 19.
a and b have common factor 19: a contradiction. Hence $\sqrt{19}$ is irrational.
Answer: Proved

5 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 - y^2 + 2y$.
Find dy/dx and d^2y/dx^2 when $y = -2$.

$dx/dy = 3y^2 - 2y + 2 = 18$ when $y = -2$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{18}$

$d^2x/dy^2 = -14$ when $y = -2$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{-14}{18^3}$

Answer: $dy/dx = \frac{1}{18}$, $d^2y/dx^2 = \frac{7}{2916}$

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6 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(58, 81^2)$. Find the value k such that $P(58 - k < X < 58 + k) = 0.95$.
Give k to 3 significant figures.

The interval is symmetrical about the mean, so each tail has probability 0.025

$P(X < 58 - k) = 0.025$, so $\frac{-k}{9} = -1.9600$

$k = 9 \times 1.9600 = 17.64$

Answer: k = 17.6

7 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x+1} - 9 \times 2^x + 4 = 0$.

Let $y = 2^x$. Then $2y^2 - 9y + 4 = 0$.

$2(y - \frac{1}{2})(y - 4) = 0$, so $y = \frac{1}{2}$ or $y = 4$.

$2^x = \frac{1}{2}$ gives $x = -1$; $2^x = 4$ gives $x = 2$.

Answer: x = −1 or x = 2

8 Trigonometric integrals with identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use a suitable trigonometric identity to find the exact value of

$\int_0^{\pi/4} \sin 3x \cos 2x \, dx$

Use $2 \sin A \cos B \equiv \sin(A + B) + \sin(A - B)$
Integrate term by term
Substitute the limits

Answer: $\frac{3}{5} - \frac{1}{5}\sqrt{2}$

9 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 40 N rests on a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{4}{3}$ and $\mu = 0.5$.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

$\sin \alpha = \frac{4}{5}$, $\cos \alpha = \frac{3}{5}$. Normal reaction: $R = 40 \cos \alpha + H \sin \alpha$

About to slide down: $H \cos \alpha + \mu R = 40 \sin \alpha$, so $H(\frac{3}{5} + 0.5 \times \frac{4}{5}) = 40(\frac{4}{5} - 0.5 \times \frac{3}{5})$

$H_{min} = 20$

About to slide up: $H \cos \alpha = 40 \sin \alpha + \mu R$, so $H(\frac{3}{5} - 0.5 \times \frac{4}{5}) = 40(\frac{4}{5} + 0.5 \times \frac{3}{5})$

$H_{max} = 220$

Answer: 20 ≤ H ≤ 220

10 Rejected batches: binomial inside binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box holds 9 components, each faulty with probability 0.15, independently. A box is rejected if it has at least 3 faulty components.
(a) Find the probability that a box is rejected.
(b) 12 boxes are inspected. Find the probability that at least 2 are rejected.

Let $X \sim B(9, 0.15)$. $P(\text{rejected}) = P(X \geq 3) = 1 - P(X \leq 2) = 0.1409$
Let Y = number of rejected boxes, $Y \sim B(12, 0.1409)$
$P(Y \geq 2) = 1 - P(Y \leq 1) = 0.520$

Answer: (a) 0.141 (b) 0.520