Day 236 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x - 6| < |-2x - 2|$.

Square both sides: $(4x - 6)^2 < (-2x - 2)^2$.
$(6x - 4)(2x - 8) < 0$.

Critical values $x = \frac{2}{3}$ and $x = 4$.

The quadratic has positive coefficient of x^2, so choose the inner region.

Answer: $\frac{2}{3} < x < 4$

2 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 recorded times (seconds) have mean 37 and standard deviation 2.
One value was recorded as 33 but should have been 35.
Find the correct mean and the correct standard deviation.

$\Sigma x = 10 \times 37 = 370$, so corrected $\Sigma x = 370 - 33 + 35 = 372$

Correct mean $= \frac{372}{10} = 37.20$

$\Sigma x^2 = 10(4 + 1369) = 13730$, corrected $\Sigma x^2 = 13730 - 1089 + 1225 = 13866$

Variance $= \frac{13866}{10} - 37.200^2 = 2.7600$, $\sigma = 1.661$

Answer: mean 37.2, standard deviation 1.66

3 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x} - 24 \times 3^x - 81 = 0$.

Let $y = 3^x$: $y^2 - 24y - 81 = 0$.
$(y - 27)(y + 3) = 0$, so $y = 27$ or $y = -3$.
$3^x > 0$ for all x, so $y = -3$ is rejected.
$3^x = 27$ gives $x = 3$.
Answer: $x = 3$

4 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 25 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially $\theta = 95$, and after 4 minutes $\theta = 60$.
Find A and k, then find when $\theta = 29$ (3 s.f.).

$t = 0$: $25 + A = 95$ so $A = 70$

$t = 4$: $25 + 70e^{-4k} = 60$ so $e^{-4k} = \frac{1}{2}$

$k = \frac{1}{4} \ln 2 = 0.173$

$70\, e^{-kt} = 4$ so $t = \frac{1}{k} \ln \frac{35}{2}$

$t = 16.5$ minutes
Answer: A = 70, k = 0.173, t = 16.5 minutes

5 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 observations x, $\Sigma(x - 20) = 10$ and $\Sigma(x - 20)^2 = 205$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 20) = \frac{10}{20} = 0.5$, so mean of $x = 20.5$

Variance $= \frac{205}{20} - (0.5)^2 = 10$

$\sigma = \sqrt{10}$

$\Sigma x^2 = \Sigma(x - 20)^2 + 40\,\Sigma(x - 20) + 20 \times 20^2$

$= 205 + 400 + 8000$

Answer: (a) 20.5 (b) 3.16 (c) 8605

6 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from level ground with horizontal and vertical velocity components 9.8 and 24.5 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take $g = 9.8$ m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 9.8
Moving upwards: $24.5 - 9.8t = 9.8$, so $t = 1.5$
Moving downwards: $24.5 - 9.8t = -9.8$, so $t = 3.5$
Horizontal distance $= 9.8 \times (3.5 - 1.5)$

Answer: (a) 1.5 s and 3.5 s (b) 19.6 m

7 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + xy - y^2 = -1$.
Find the equation of the tangent to C at the point $(-3, 2)$,
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At $(-3, 2)$: $dy/dx = -\frac{4}{7}$

$y - 2 = (-\frac{4}{7})(x - -3)$, then clear fractions

Answer: $4x + 7y - 2 = 0$

8 Trigonometric integrals with identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use a suitable trigonometric identity to find the exact value of

$\int_0^{\pi/3} \sin 4x \cos x \, dx$

Use $2 \sin A \cos B \equiv \sin(A + B) + \sin(A - B)$
Integrate term by term
Substitute the limits

Answer: $\frac{23}{60}$

9 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−2, 5), B(1, 6)
and C(−4, 1). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of AB: $3x + y = 4$.
Bisector of BC: $-5x - 5y = -10$.
Solving: centre (1, 1).
$r^2 = (-2 - 1)^2 + (5 - 1)^2 = 25$.

Answer: $(x - 1)^2 + (y - 1)^2 = 25$

10 Block on a plank: limiting slip (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A plank of mass 5 kg lies on a smooth horizontal table, with a block of mass 5 kg on it ($\mu = 0.4$ between them). A horizontal force P N acts on the plank.

(a) Find the greatest P for which the block does not slip. (b) Find the accelerations of block and plank when P = 117.6. Take $g = 9.8$ m s^{-2}.

No slipping: common acceleration $a = \frac{P}{10}$. The block is accelerated only by friction: $F = 5a \leq \mu mg$

So $a \leq \mu g = 3.92$, and P_max = 10 × 3.92 = 39.2

P = 117.6 > 39.2, so the block slips. Block: $a = \mu g = 3.92$

Plank: 117.6 − 19.6 = 5 a, so a = 19.6

Answer: (a) 39.2 N (b) block 3.92 m s^{-2}, plank 19.6 m s^{-2}