Day 232 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 175 N rests on a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$ and $\mu = 0.5$.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

$\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. Normal reaction: $R = 175\cos\alpha + H\sin\alpha$

About to slide down: $H\cos\alpha + \mu R = 175\sin\alpha$, so $H(\frac{3}{5} + 0.5 \times \frac{4}{5}) = 175(\frac{4}{5} - 0.5 \times \frac{3}{5})$

$H_{min} = 87.5$

About to slide up: $H\cos\alpha = 175\sin\alpha + \mu R$, so $H(\frac{3}{5} - 0.5 \times \frac{4}{5}) = 175(\frac{4}{5} + 0.5 \times \frac{3}{5})$

$H_{max} = 962.5$

Answer: $87.5 \leq H \leq 962.5$

2 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $e^{2x} - 6 \times e^{x} + 5 = 0$,
giving each answer exactly and, where not an integer, to 3 s.f.

Let $u = e^x$: $u^2 - 6u + 5 = 0$

$(u - 5)(u - 1) = 0$, so $e^x = 5$ or $e^x = 1$

Take logarithms: $x = \ln 1 = 0$; $x = \ln 5 = 1.61$

Answer: $x = \ln 1 = 0$; $x = \ln 5 = 1.61$

3 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(58, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find $P(X > 74)$.

The area 0.6 leaves 0.400 in total in the tails, 0.200 in each

$P(X < 47) = 0.200$ so $\frac{47 - 58}{\sigma} = -0.8416$

$\sigma = \frac{11}{0.8416} = 13.07$

$P(X > 74) = P(Z > 1.224) = 0.110$

Answer: (a) 13.1 (b) 0.110

4 Newton-Raphson iteration (AQA, Edexcel, OCR, MEI, Eduqas (Newton-Raphson))

$f(x) = x^3 + x^2 - 3x - 14$.
(a) Show that $f(x) = 0$ has a root α with $2 < \alpha < 3$.
(b) Use Newton-Raphson with $x_0 = 2$ to find x_1 and x_2
to 4 decimal places.

$f(2) = -8$ and $f(3) = 13$: a change of sign, f continuous

$f'(x) = 3x^2 + 2x - 3$, so $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \frac{-8.000}{13.00} = 2.6154$

$x_2 = 2.6154 - \frac{2.884}{22.75} = 2.4886$

Answer: $x_1 = 2.6154$, $x_2 = 2.4886$

5 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 2 and its common difference d is non-zero. The 2nd, 4th and 8th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: $2 + d$, $2 + 3d$, $2 + 7d$.

Geometric: $(2 + 3d)^2 = (2 + d)(2 + 7d)$.

Expand: $9d^2 + 12d + 4 = 7d^2 + 16d + 4$, so $d(2d - 4) = 0$.

$d \neq 0$, so $d = 2$. Ratio $= \frac{8}{4} = 2$.

Answer: $d = 2$, ratio $= 2$

6 Range and greatest height to speed and angle (AQA, Edexcel, OCR, MEI, Eduqas)

A golf ball is hit from a point O on horizontal ground. It lands 24 m from O, and reaches a maximum height of 6 m above the ground.

Find the angle of projection and the speed of projection, to 3 s.f. Take $g = 9.8$ m s^{-2}.

Vertical: $0 = u_y^2 - 2 \times 9.8 \times 6$, so $u_y = 10.84$

Time of flight $= \frac{2u_y}{9.8} = 2.213$ s, so $u_x = \frac{24}{2.213} = 10.84$

$\tan\theta = \frac{u_y}{u_x} = 1.000$, so $\theta = 45.0°$

Speed $= \sqrt{(u_x^2 + u_y^2)} = 15.3$

Answer: $\theta = 45.0°$, speed 15.3 m s^{-1}

7 Binomial approximation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(8 - x)^{\frac{1}{3}}$ up to and including the term in x^2.

(b) Use $x = 0.02$ to estimate $\sqrt[3]{7.98}$ to 5 decimal places.

(a) $(8 - x)^{\frac{1}{3}} = 2(1 - \frac{1}{8}x)^{\frac{1}{3}}$.

$= 2 - \frac{1}{12}x - \frac{1}{288}x^2 + ...$

(b) $8 - x$ with $x = 0.02$ gives 7.98.

Substitute: $2 + (-\frac{1}{12})(0.02) + (-\frac{1}{288})(0.02)^2$.

$= 1.99833$ (5 d.p.).

Answer: (a) $2 - \frac{1}{12}x - \frac{1}{288}x^2$ (b) 1.99833

8 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle with equation $(x - 1)^2 + (y + 3)^2 = 169$
passes through P(13, 2). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre $(1, -3)$. Gradient of radius CP $= \frac{5}{12}$.

Tangent gradient $= -\frac{12}{5}$: $12x + 5y = 166$.

$x = 0$: $y = \frac{166}{5}$. $y = 0$: $x = \frac{83}{6}$.

Area $= ½ \times \frac{83}{6} \times \frac{166}{5}$.

Answer: $\frac{6889}{30}$

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9 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(120, 0.45).

(a) State why a normal approximation is suitable for X.

(b) Use a normal approximation to estimate P(49 ≤ X ≤ 59), to 3 significant figures.

np = 54 and nq = 66, both greater than 5 (n large, p not extreme)

Y ~ N(54, 29.7) with σ = 5.450

Apply the continuity correction:

P(49 ≤ X ≤ 59) ≈ P(48.5 < Y < 59.5) = 0.8436 − 0.1564 = 0.687

Answer: 0.687

10 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-x-2}{(x+3)^2}$ in the form $\frac{A}{x+3}+\frac{B}{(x+3)^2}$.

(b) Find the exact value of $\int_0^3 \frac{-x-2}{(x+3)^2}\,dx$.

$-x-2 \equiv A(x+3)+B$, so $A=-1$, and $B=1$

Integral = $[-1\ln(x+3)-\frac{1}{x+3}]$

Substitute limits 0 and 3

Answer: (a) $A=-1$, $B=1$; (b) $\frac{1}{6}-\ln 2$