Day 230 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{6 - 3\sqrt{2}}{4 - 3\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(4 + 3\sqrt{2})$.
Numerator: $(6 - 3\sqrt{2})(4 + 3\sqrt{2}) = 6 + 6\sqrt{2}$.
Denominator: $4^2 - 3^2(2) = -2$.
So the fraction = $\frac{6 + 6\sqrt{2}}{-2}$.

Answer: $-3 - 3\sqrt{2}$

2 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of
$\tan(\tan^{-1}(\frac{2}{5}) + \tan^{-1}(1))$.

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
$= \frac{\frac{2}{5} + 1}{1 - \frac{2}{5} \times 1} = \frac{\frac{7}{5}}{\frac{3}{5}}$

Answer: $\frac{7}{3}$

3 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x + 2} - 10 \times 3^x + 1 = 0$.

Let $y = 3^x$. Then $9y^2 - 10y + 1 = 0$.
$9(y - \frac{1}{9})(y - 1) = 0$, so $y = \frac{1}{9}$ or $y = 1$.
$3^x = \frac{1}{9}$ gives $x = -2$; $3^x = 1$ gives $x = 0$.

Answer: $x = -2$ or $x = 0$

4 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact gradient of the curve $y = 2^{4x - 2}$
at the point where $x = \frac{3}{2}$.

$y = e^{(4x - 2)\ln 2}$
$dy/dx = 4 \ln 2 \times 2^{4x - 2}$
At $x = \frac{3}{2}$: exponent = 4, so $dy/dx = 4 \ln 2 \times 2^4$

Answer: $64\ln 2$

5 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through P(9, −9), Q(−3, −15)
and R(12, 0). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of PQ: $-12x - 6y = 36$.
Bisector of QR: $15x + 15y = -45$.
Solving: centre $(-3, 0)$.
$r^2 = (9 - (-3))^2 + (-9 - 0)^2 = 225$.

Answer: $(x + 3)^2 + (y + 0)^2 = 225$

6 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.25, P(B) = 0.4 and P(A | B) = 0.5.
(a) Find P(A ∩ B).
(b) Find P(A | B′).

P(A ∩ B) = P(A | B) × P(B) = 0.5 × 0.4 = 0.2
P(A ∪ B) = 0.25 + 0.4 − 0.2 = 0.45
P(B′) = 0.6
P(A ∩ B′) = P(A) − P(A ∩ B) = 0.05
$P(A \mid B') = \frac{0.05}{0.6}$

Answer: (a) 0.2 (b) $\frac{1}{12}$

7 Two particles over a pulley: height reached (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 12 kg and 6 kg hang on a light string over a smooth fixed pulley. The 12 kg particle is 3.75 m above the ground; the 6 kg one is on the ground.
Released from rest, find (a) the tension, (b) the greatest height reached by the 6 kg particle. Take g = 9.8 m s^{-2}.

$12\,g - T = 12\,a$ and $T - 6\,g = 6\,a$, so $a = \frac{12 - 6}{12 + 6} \times 9.8 = \frac{49}{15}$

$T = 12(9.8 - \frac{49}{15}) = 78.4$

The 6 kg particle rises 3.75 m before the 12 kg particle lands, reaching $v^2 = 2 \times \frac{49}{15} \times 3.75 = 24.5$

The string goes slack, so it then moves freely under gravity: extra height = $\frac{v^2}{2g} = \frac{24.5}{19.6} = 1.25$

Greatest height = 3.75 + 1.25
Answer: (a) 78.4 N (b) 5 m

8 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 6 m and mass 30 kg rests on supports at C and D, where AC = 2 m and DB = 2 m. A girl of mass 40 kg stands on it at distance x m from A.
Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs $R_C \geq 0$ and $R_D \geq 0$. Limiting cases: $R_C = 0$ and $R_D = 0$ in turn

Tilting about D (girl beyond D): $30(1) = 40(x - 4)$, so $x = 4 + \frac{30 \times 1}{40} = 4.75$

Tilting about C (girl before C): $30(1) = 40(2 - x)$, so $x = 2 - \frac{30 \times 1}{40} = 1.25$

Answer: $1.25 \leq x \leq 4.75$

9 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: p = 0.4 against H_1: p > 0.4 uses a sample of 40 at the 5% level.
(a) Find the critical region for X, the number of successes.
(b) Find the actual significance level.
(c) Find P(Type II error) when in fact p = 0.55.

X ~ B(40, 0.4). P(X ≥ 22) = 0.0392 and P(X ≥ 21) = 0.0744
The first is at most 0.05 and the second is not, so the critical region is X ≥ 22
Actual significance level = P(X ≥ 22) = 0.0392
Type II error: H_0 is not rejected, so X ≤ 21, with X ~ B(40, 0.55)
P(X ≤ 21) = 0.435
Answer: (a) X ≥ 22 (b) 0.0392 (c) 0.435

Day 230 **A Level Maths: S Tier 10-a-day: Model Solutions**

10 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the points of inflection of

$y = \frac{2}{x^2 + 12}$.

$\frac{dy}{dx} = -\frac{4x}{(x^2 + 12)^2}$

$\frac{d^2y}{dx^2} = \frac{4(3x^2 - 12)}{(x^2 + 12)^3}$

Zero when $x^2 = 4$, so $x = \pm 2$, and the sign changes either side

$y = \frac{2}{16} = \frac{1}{8}$

Answer: $(2, \frac{1}{8})$ and $(-2, \frac{1}{8})$