Day 225 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = 3t$, $y = 3t^2$. The line $y = -3x + 12$
meets C at A and B. Find the exact length AB.

Substitute: $3t^2 = -9t + 12$.
$3t^2 + 9t - 12 = 0 \Rightarrow t = 1$ or $t = -4$.
A(3, 3), B(−12, 48).
$AB^2 = 15^2 + (-45)^2 = 2250$.
Answer: AB = $15\sqrt{10}$

2 Normal: find mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times taken by a bus, in minutes are normally distributed. 20% are less than 30 and 20% are greater than 54. Find the mean and standard deviation, each correct to 3 significant figures.

Let $X \sim N(\mu, \sigma^2)$. $P(X < 30) = 0.20$ gives $\frac{30 - \mu}{\sigma} = -0.8416$

$P(X > 54) = 0.20$ gives $\frac{54 - \mu}{\sigma} = 0.8416$

Subtract: $\frac{24}{\sigma} = 0.8416 - (-0.8416) = 1.6832$
$\sigma = 14.26$
$\mu = 30 - (-0.8416)(14.26) = 42.00$
Answer: μ = 42.0, σ = 14.3

3 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x + y = 1$
$2x^2 - 2xy - y^2 = 2$

From the first equation $x = 1 - y$.
Substitute into the second equation and collect terms:
this gives $3y^2 - 6y = 0$.
$y = 0$ or $y = 2$, giving $x = 1$ or $x = -1$.
Answer: (x, y) = (1, 0) and (−1, 2)

4 R-form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.
(b) Hence solve $4\cos\theta + 3\sin\theta = -1$ for $0 \leq \theta < 360°$, giving answers to 1 d.p.

$R = \sqrt{4^2 + 3^2} = 5$ and $\tan\alpha = \frac{3}{4}$, so $\alpha = 36.9°$

So $4\cos\theta + 3\sin\theta \equiv 5\cos(\theta - 36.9°)$

$\cos(\theta - \alpha) = \frac{-1}{5}$

$\theta - \alpha = \pm 101.5°$ (using $\alpha = 36.9°$)
Adding or subtracting multiples of 360° to give $0 \leq \theta < 360°$
Answer: (a) R = 5, α = 36.9° (b) θ = 138.4°, 295.3°

5 Slide down a rough slope then along rough ground (CIE)

A particle starts from rest at the top of a rough slope of length 15 m ($\sin\alpha = \frac{4}{5}$, $\mu = 0.6$), then slides along rough horizontal ground with the same μ.
Use energy to find (a) its speed at the foot of the slope (3 s.f.), (b) the distance it slides on the ground. Take $g = 9.8$ m s^{-2}.

On the slope: $\frac{1}{2} m v^2 = m g \times 15 \sin\alpha - \mu m g \cos\alpha \times 15$

$v^2 = 2 \times 9.8 \times 15(\frac{4}{5} - 0.6 \times \frac{3}{5}) = 129.36$, so $v = 11.37$

On the ground: $\frac{1}{2} m v^2 = \mu m g x$, so $x = \frac{v^2}{2\mu g} = \frac{129.36}{11.76} = 11$

Answer: (a) 11.4 m s^{-1} (b) 11 m

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2y + xy^2 = -12$.
Find the equation of the normal to C at the point $(4, -3)$,
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At $(4, -3)$: $dy/dx = -\frac{15}{8}$

Normal gradient $= -1 \div (-\frac{15}{8}) = \frac{8}{15}$

$y - -3 = \frac{8}{15}(x - 4)$, then clear fractions

Answer: 8x − 15y − 77 = 0

7 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{6x + 7} = 2x - 1$.

Square both sides: $6x + 7 = (2x - 1)^2$.

$4x^2 - 10x - 6 = 0$, so $x = -\frac{1}{2}$ or $x = 3$.

Check in $\sqrt{6x + 7} = 2x - 1$: $x = 3$ gives $5 \geq 0$ (valid).

$x = -\frac{1}{2}$ gives right-hand side $-2 < 0$, so it is rejected.

Answer: x = 3

8 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 10 + Ae^{-kt}$, where θ °C is the temperature
after t minutes. Initially $\theta = 100$, and after 10 minutes $\theta = 73$.
Find A and k, then find when $\theta = 48$ (3 s.f.).

$t = 0$: $10 + A = 100$ so $A = 90$

$t = 10$: $10 + 90e^{-10k} = 73$ so $e^{-10k} = \frac{7}{10}$

$k = \frac{1}{10} \ln \frac{10}{7} = 0.0357$

$90\, e^{-kt} = 38$ so $t = \frac{1}{k} \ln \frac{45}{19}$

$t = 24.2$ minutes

Answer: A = 90, k = 0.0357, t = 24.2 minutes

9 Just clearing a wall at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 45° to the horizontal. The ball just passes over the top of a vertical wall of height 10.8 m, at a horizontal distance of 99 m from O.

Find (a) the speed of projection, (b) the greatest height of the ball above the ground, to 3 s.f. Take g = 9.8 m s^{-2}.

At 45°: $x = \frac{u\,t}{\sqrt{2}}$ and $y = \frac{u\,t}{\sqrt{2}} - 4.9t^2$, so $y = x - \frac{9.8x^2}{u^2}$

$10.8 = 99 - \frac{9.8 \times 99^2}{u^2}$, so $u^2 = \frac{9.8 \times 9801}{88.2} = 1089$

$u = 33$

Greatest height $= \frac{(u \sin 45°)^2}{(2 \times 9.8)} = \frac{1089}{39.2}$

Answer: (a) 33 m s^{-1} (b) 27.8 m

10 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent.

Find x and y.

P(A) = 1x + x = 2x, P(B) = x + y, $P(A \cap B) = x$

Independent: $x = 2x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{2}$

Total probability: $x + x + y + 0.3 = 1$, so $x + (x + y) = 0.7$

$x = 0.7 - \frac{1}{2} = 0.2$

$x = \frac{1}{5}$, $y = \frac{1}{2} - x = \frac{3}{10}$

Answer: $x = \frac{1}{5}$, $y = \frac{3}{10}$