Day 223 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_3(x + 14) + \log_3(x - 4) = 5$,
justifying clearly any root that you reject.

$(x + 14)(x - 4) = 3^5 = 243$
$x^2 + 10x - 299 = 0$
$(x - 13)(x + 23) = 0$, so $x = 13$ or $x = -23$
$x = -23$ makes $x + 14 = -9 < 0$ and log of a negative number is undefined
Answer: x = 13

2 Waiting for the r-th success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot Amira takes scores with probability $\frac{1}{2}$, independently.
(a) Find the probability that her fourth score comes on shot number 8.
(b) Find the probability that her fourth score comes on or before shot number 8.

(a) fourth score on shot 8: exactly 3 scores in the first 7 shots, then a score
$C(7, 3)(\frac{1}{2})^3(\frac{1}{2})^4 \times \frac{1}{2}$
$= 0.137$

(b) On or before shot 8 means at least 4 scores in 8 shots: $X \sim B(8, \frac{1}{2})$
$P(X \geq 4) = 1 - P(X \leq 3) = 0.637$
Answer: (a) 0.137 (b) 0.637

3 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $6\sec^2\theta - 16\tan\theta + 2 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $6\tan^2\theta - 16\tan\theta + 8 = 0$
$2(3\tan\theta - 2)(\tan\theta - 2) = 0$
$\tan\theta = \frac{2}{3}$ or $\tan\theta = 2$
$\theta = 33.7°$ and $63.4°$ $(+180°)$
Answer: θ = 33.7°, 63.4°, 213.7°, 243.4°

4 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-1 + 5\sqrt{6}}{2 + \sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(2 - \sqrt{6})$.
Numerator: $(-1 + 5\sqrt{6})(2 - \sqrt{6}) = -32 + 11\sqrt{6}$.
Denominator: $2^2 - 1^2(6) = -2$.
So the fraction = $\frac{-32 + 11\sqrt{6}}{-2}$.

Answer: $16 - \frac{11}{2}\sqrt{6}$

5 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from level ground with horizontal and vertical velocity components 34.3 and 39.2 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 34.3$
Moving upwards: $39.2 - 9.8t = 34.3$, so $t = 0.5$
Moving downwards: $39.2 - 9.8t = -34.3$, so $t = 7.5$
Horizontal distance = $34.3 \times (7.5 - 0.5)$
Answer: (a) 0.5 s and 7.5 s (b) 240.1 m

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = \ln t$, $y = 5t^2 + 5t$, $t > 0$.
Find the equation of the tangent where $t = 1$, in the form $y = mx + c$.

$\frac{dx}{dt} = \frac{1}{t}$, $\frac{dy}{dt} = 10t + 5$

$\frac{dy}{dx} = t(10t + 5)$ which is 15 at $t = 1$
At $t = 1$: $x = 0$, $y = 10$
Answer: $y = 15x + 10$

7 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ab^x$. The graph of lg y against x is the straight line shown.
Find a and b to 3 significant figures.

$\lg y = \lg a + x \lg b$. The intercept is $\lg a = 0.5$, so $a = 10^{0.5} = 3.162$

Gradient $= \frac{1.3 - 0.5}{8} = 0.1$

$\lg b = 0.1$ so $b = 10^{0.1} = 1.259$
Answer: a = 3.16, b = 1.26

8 Tangent line to circle: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $5y = 12x + k$ is a tangent to the circle
$(x + 0)^2 + (y - 2)^2 = 4$.
Find the two possible values of k.

Substitute $y = \frac{12x + k}{5}$: $(x + 0)^2 + (\frac{12x + k}{5} - 2)^2 = 4$.

Multiply by 25 and collect: a quadratic in x with $a = 169$.

Tangent $\Rightarrow$ discriminant = 0. This simplifies to $(k - 10)^2 = 676$.
$k - 10 = \pm 26$.
Answer: $k = -16$ or $k = 36$

9 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (3 m, 120 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan \alpha = 0.75$, where α is the angle between string and rod. A 30 N particle hangs at P, AP = 2.25 m.
Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin \alpha = \frac{3}{5}$, $\cos \alpha = \frac{4}{5}$. Moments about A: $T \sin \alpha \times 3 = 120 \times 1.5 + 30 \times 2.25$

$T \times \frac{3}{5} \times 3 = 247.5$, so $T = 137.5$

Horizontal: $H = T \cos \alpha = 110$. Vertical: $V + T \sin \alpha = 120 + 30$, so $V = 67.5$

Force $= \sqrt{(110^2 + 67.5^2)} = 129$, angle $= \tan^{-1}(\frac{67.5}{110}) = 31.5°$ above the horizontal

Answer: (a) 137.5 N (b) 129 N at 31.5° above the horizontal (away from the wall)

10 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 6 and its common difference d is non-zero. The 4th, 9th and 19th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: $6 + 3d$, $6 + 8d$, $6 + 18d$.

Geometric: $(6 + 8d)^2 = (6 + 3d)(6 + 18d)$.

Expand: $64d^2 + 96d + 36 = 54d^2 + 126d + 36$, so $d(10d - 30) = 0$.

$d \neq 0$, so $d = 3$. Ratio $= \frac{30}{15} = 2$.

Answer: d = 3, ratio = 2