Day 222 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass M kg, carrying a passenger of mass 90 kg, hangs from a vertical cable. The tension in the cable is 3354 N and the floor exerts a force of 774 N on the passenger.

Find (a) the acceleration of the lift, with its direction, (b) the value of M. Take g = 9.8 m s^{-2}.

Passenger (upwards positive): $774 - 90 \times 9.8 = 90a$, so $a = -1.2$

$a = 1.2$ m s^{-2}, so the acceleration is downwards

Lift and passenger: $3354 - (M + 90) \times 9.8 = (M + 90) \times -1.2$

$(M + 90)(9.8 - 1.2) = 3354$, so $M + 90 = \frac{3354}{8.6} = 390$

Answer: (a) 1.2 m s^{-2} downwards (b) M = 300

2 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos(\theta + 25°) = \frac{\sqrt{3}}{2}$

for $-90° \leq \theta \leq 270°$.

Let $u = \theta + 25°$; then $\cos u = \frac{\sqrt{3}}{2}$

Base solutions: $u = 30, -30°$ (+ multiples of 360°)

$\theta = (u - 25°) \div 1$, keeping those in range

Answer: $\theta = -55°, 5°$

3 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of

$y = 3f(2x + 8) - 3$.

Inside f: replace x by $2x + 8$, so x-coordinates become $(x - 8) \div 2$.

Outside: y-coordinates become $3 \times y - 3$.

A: $(-7, 9)$ B: $(-3, 0)$

C: $(-2, 9)$ D: $(-1, -3)$

Answer: A′(−7, 9), B′(−3, 0), C′(−2, 9), D′(−1, −3)

4 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A javelin is projected from a point O on horizontal ground with speed 63.7 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{12}{5}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 24.5, vertical 58.8 (as $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$)

Time of flight: $0 = 58.8t - 4.9t^2$, so $t = \frac{58.8}{4.9} = 12$

Range $= 24.5 \times 12 = 294$

Greatest height: $0 = 58.8^2 - 2 \times 9.8 \times H$, so $H = \frac{3457.44}{19.6} = 176.4$

Answer: (a) 12 s (b) 294 m (c) 176.4 m

5 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both

$x^2 - 1 \geq 0$ and $-x - 8 \leq 1$.

Quadratic: $(x + 1)(x - 1) \geq 0$ gives $x \leq -1$ or $x \geq 1$.

Linear: $-x - 8 \leq 1$ gives $x \geq -9$. (the inequality reverses as we divide by a negative)

Both must hold: take the overlap of the two sets.

Answer: $-9 \leq x \leq -1$ or $x \geq 1$

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6 Sum of two independent copies of a discrete variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X takes values 2, 3, 4, 5 with $P(X = x) = k(x - 1)$. X_1 and X_2 are independent copies of X.

(a) Find k.

(b) Find $P(X_1 + X_2 = 6)$.

(c) Find $P(X_1 = X_2)$ and $E(X_1 + X_2)$.

(a) $\Sigma P = k(1 + 2 + 3 + 4) = 10\,k = 1$, so $k = \frac{1}{10}$

(b) Ways to total 6: (2,4), (3,3), (4,2)

$P = \frac{1}{10}\times\frac{3}{10} + \frac{1}{5}\times\frac{1}{5} + \frac{3}{10}\times\frac{1}{10} = \frac{1}{10}$

(c) $P(X_1 = X_2) = \Sigma\,[P(X = x)]^2 = \frac{3}{10}$

$E(X) = 4$, so $E(X_1 + X_2) = 2 \times 4 = 8$

Answer: (a) $k = \frac{1}{10}$ (b) $\frac{1}{10}$ (c) $\frac{3}{10}$, 8

7 Integration by substitution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = e^x + 3$ to find the exact value of

$$\int_0^{\ln 3} \frac{e^x}{e^x + 3}\,dx$$

$du = e^x\,dx$; limits $x = 0$, $\ln 3$ become $u = 4, 6$

Integral $= \int_4^6 \frac{1}{u}\,du = [\ln u]$

Evaluate the limits

Answer: $-\ln 2 + \ln 3$

8 Alternate turns and sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B take turns, A first, to score; the first to score wins. P(A scores) = $\frac{2}{5}$, P(B scores) = $\frac{1}{3}$ each attempt.

Find P(A wins (a) on attempt 1 or 2, (b) overall).

P(A wins at attempt 1) = $\frac{2}{5}$; to reach attempt 2 both miss first: $(\frac{3}{5})(\frac{2}{3}) = \frac{2}{5}$

(a) $\frac{2}{5} + \frac{2}{5} \times \frac{2}{5} = \frac{14}{25}$

(b) $P(A\text{ wins}) = \frac{2}{5} + \frac{2}{5} \times \frac{2}{5} + \frac{2}{5}^2 \times \frac{2}{5} + \ldots$

Geometric: first term $\frac{2}{5}$, common ratio $\frac{2}{5}$

$S = \frac{a}{1 - r} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$

Answer: (a) $\frac{14}{25}$ (b) $\frac{2}{3}$

9 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\sin(\sin^{-1}(\frac{5}{13}) + \cos^{-1}(\frac{7}{25}))$.

Let $A = \sin^{-1}(\frac{5}{13})$ and $B = \cos^{-1}(\frac{7}{25})$, both acute

$\sin A = \frac{5}{13}$, $\cos A = \frac{12}{13}$, $\cos B = \frac{7}{25}$, $\sin B = \frac{24}{25}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{5}{13} \times \frac{7}{25} + \frac{12}{13} \times \frac{24}{25}$

Answer: $\frac{323}{325}$

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10 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{4x-2}{-2x+5} > 2$.

Rearrange: $\frac{4x-2}{-2x+5} - 2 > 0$.

Combine: $\frac{8x-12}{-2x+5} > 0$.

Critical values: $x = \frac{3}{2}$ (zero of numerator), $x = \frac{5}{2}$ (excluded, denominator zero).

Sign analysis of the three regions gives the solution.

Answer: $\mathbf{\frac{3}{2} < x < \frac{5}{2}}$