**Day 220** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-3x^2 - 14x + 22}{(x-2)(3x-3)^2}$ in the form

$\frac{A}{(x-2)} + \frac{B}{(3x-3)} + \frac{C}{(3x-3)^2}$.

$-3x^2 - 14x + 22 \equiv A(3x-3)^2 + B(x-2)(3x-3) + C(x-2)$.

Put $x = 2$: $-18 = A(3)^2$, so $A = -2$.

Put $x = 1$: $5 = C(-1)$, so $C = -5$.

Compare $x^2$: $-3 = 9A + 3B$, so $B = 5$.

**Answer: A = −2, B = 5, C = −5**

## 2 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean air temperature (°C), Heathrow, October 2015 (n = 184): mean 14.4, sd 3.1. ($F = 1.8C + 32$)

(a) Find the mean and sd in °F.

(b) Is a reading of 20 more than 2 sd above the mean?

New value = 1.8 × old + 32, so mean = 1.8 × 14.4 + 32 = 57.92

The constant shifts the mean but not the spread; only the multiplier affects σ

σ = 1.8 × 3.1 = 5.580

Outlier cut-off = mean + 2σ = 14.4 + 6.2 = 20.6 (original units)

20 is less than 20.6, so no

**Answer: (a) mean 57.9, sd 5.58 (b) no**

## 3 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m $s^{-1}$, time in s). At time 9 s, P is back at its starting point.

Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 3 × 12 + ½(12 − W)(4) − W(2) = 0

18 + 24 − 2W − 2W = 0, so 4W = 42

W = 10.5

v = 0 where the line from (3, 12) to (7, −10.5) crosses the axis: $t = 3 + 4 \times \frac{12}{22.5} = \frac{77}{15}$

Greatest distance = area of triangle = $\frac{1}{2} \times \frac{77}{15} \times 12$

**Answer: (a) W = 10.5 (b) 30.8 m**

## 4 Tangent line to circle: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $5y = 12x + k$ is a tangent to the circle

$(x+3)^2 + (y-3)^2 = 1$.

Find the two possible values of k.

Substitute $y = \frac{12x + k}{5}$: $(x+3)^2 + (\frac{12x + k}{5} - 3)^2 = 1$.

Multiply by 25 and collect: a quadratic in x with a = 169.

Tangent ⇒ discriminant = 0. This simplifies to $(k - 51)^2 = 169$.

$k - 51 = \pm 13$.

**Answer: k = 38 or k = 64**

## 5 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{2x + 12}$ at the point where $x = 2$ meets the axes at A and B. Find the exact area of triangle OAB, where O is the origin.

At $x = 2$: $y = 8$. Differentiate: gradient $= \frac{9}{2}$

Tangent: $y - 8 = \frac{9}{2}(x - 2)$, so $y = \frac{9}{2}x - 1$

y-intercept $-1$; x-intercept $\frac{2}{9}$

Area $= \frac{1}{2} \times |-1| \times |\frac{2}{9}|$

**Answer:** $\frac{1}{9}$

## 6 Proof by contradiction (irrationality) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sqrt{10}$ is irrational, prove by contradiction that $\sqrt{5} + \sqrt{2}$ is irrational.

Assume $\sqrt{5} + \sqrt{2} = r$, where r is rational.

Square both sides: $5 + 2 + 2\sqrt{10} = r^2$.

So $\sqrt{10} = \frac{r^2 - 7}{2}$.

The right-hand side is rational, but $\sqrt{10}$ is irrational.

This is a contradiction, so $\sqrt{5} + \sqrt{2}$ is irrational.

**Answer: Proved**

## 7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a parcel is $N(64, 5^2)$ kg. A parcel is 'heavy' if it exceeds 69.

(a) Find the probability that a parcel is heavy.

(b) 17 parcels are chosen at random. Find the probability that at least 3 are heavy.

(a) $P(X > 69) = P(Z > 1.000) = 0.1587$

(b) Let Y = number heavy, $Y \sim B(17, 0.1587)$

$P(Y \geq 3) = 1 - P(Y \leq 2) = 0.520$

**Answer: (a) 0.159 (b) 0.520**

## 8 Area between a curve and a line (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curves $y = 5\cos x$ and $y = 5\cos 2x$ meet at $x = 0$ and at $x = \frac{2\pi}{3}$. Find the exact area enclosed between them for $0 \leq x \leq \frac{2\pi}{3}$.

For $0 < x < \frac{2\pi}{3}$, $\cos x > \cos 2x$

Area $= 5\int_0^{2\pi/3} (\cos x - \cos 2x)\, dx = 5[\sin x - \frac{\sin 2x}{2}]$

$= 5[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}]$

**Answer:** $\frac{15}{4}\sqrt{3}$

## 9 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 4 m $s^{-1}$ up a line of greatest slope of a rough plane inclined at $\alpha$ to the horizontal, where $\sin\alpha = \frac{3}{5}$ and $\mu = 0.35$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m $s^{-2}$.

$\cos\alpha = \frac{4}{5}$. Up the plane: $R = mg\cos\alpha$, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{3}{5} + 0.35 \times \frac{4}{5}) = 8.624$

$0 = 4^2 - 2 \times 8.624 \times d$, so $d = \frac{16}{17.248} = 0.9276$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{3}{5} - 0.35 \times \frac{4}{5}) = 3.136$

$v^2 = 2 \times 3.136 \times 0.92764 = 5.818$, so $v = 2.41$

**Answer: (a) 0.928 m (b) 2.41 m $s^{-1}$**

## 10 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two boats P and Q move so that, t hours after noon, their position vectors in km are

$\mathbf{r}_P = \begin{pmatrix}-3\\3\end{pmatrix} + t\begin{pmatrix}0\\3\end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix}-12\\9\end{pmatrix} + t\begin{pmatrix}3\\3\end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}-9\\6\end{pmatrix} + t\begin{pmatrix}3\\0\end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix}3\\0\end{pmatrix} = 0$

$-27 + t(9) = 0$, so $t = 3$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}0\\6\end{pmatrix}$, with length $\sqrt{36}$

**Answer: t = 3, minimum distance = 6 km**