Day 219 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\sec^2\theta - \tan\theta - 3 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta - \tan\theta - 1 = 0$
$(\tan\theta - 1)(2\tan\theta + 1) = 0$
$\tan\theta = 1$ or $\tan\theta = -\frac{1}{2}$
$\theta = 45.0°$ and $153.4°$ $(+180°)$
Answer: θ = 45.0°, 153.4°, 225.0°, 333.4°

2 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 4 = 0$ differ by 3.
Find the possible values of k.

Let the roots be α and $\alpha + 3$.
Product: $\alpha(\alpha + 3) = 4$, so $\alpha^2 + 3\alpha - 4 = 0$.
$(\alpha - 1)(\alpha + 4) = 0$, so $\alpha = 1$ or $\alpha = -4$.
Sum: $2\alpha + 3 = -k$, so $k = -(2\alpha + 3) = -5$ or 5.
Answer: k = ±5

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from a point O on horizontal ground with speed 49 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 29.4^2 - 2 \times 9.8 \times H$, so $H = 44.1$
At height $\frac{H}{2} = 22.05$: $v_y^2 = 29.4^2 - 2 \times 9.8 \times 22.05 = 432.18$
$v_x = 39.2$ (constant), so speed^2 = 1536.64 + 432.18 = 1968.82
Answer: (a) 44.1 m (b) 44.4 m s^{-1}

4 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic y = f(x), which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at x = 1 and touches at x = −3, so $f(x) = k(x - 1)(x + 3)^2$.
At (0, −9): $-9 = k(-1)(3)^2 = -9k$, so $k = 1$.
$f(x) = (x - 1)(x + 3)^2$, then expand.
Answer: $f(x) = x^3 + 5x^2 + 3x - 9$

5 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 14.7 and 63.7 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 14.7$
Moving upwards: $63.7 - 9.8t = 14.7$, so $t = 5$
Moving downwards: $63.7 - 9.8t = -14.7$, so $t = 8$
Horizontal distance = $14.7 \times (8 - 5)$
Answer: (a) 5 s and 8 s (b) 44.1 m

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6 Least number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each battery is faulty with probability 0.12, independently.
Find the least number n of batterys that must be tested so that the probability of finding at least one faulty is greater than 0.95.

P(at least one faulty) = $1 - 0.88^n > 0.95$
$0.88^n < 0.05$
$n \ln 0.88 < \ln 0.05$
Dividing by ln 0.88 (negative) reverses the inequality: $n > 23.43$
$n = 24$
Answer: n = 24

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x^3\sqrt{175 - x}$, for $0 < x < 175$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{2x^2(1050 - 7x)}{2\sqrt{175 - x}}$
$\frac{dy}{dx} = 0$: $1050 - 7x = 0$, so $x = 150$
$y = 2 \times 150^3 \times 5$
Gradient changes from positive to negative: a maximum
Answer: (150, 33750000), maximum

8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{2}{18 + 8\sin\theta + 15\cos\theta}$
(a) Write $8\sin\theta + 15\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.
(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{8^2 + 15^2} = 17$, $\tan\alpha = \frac{15}{8}$, $\alpha = 61.9°$
Denominator = $18 + 17\sin(\theta + 61.9°)$, which lies between 1 and 35
Greatest $f(\theta) = \frac{2}{1} = 2$, when $\sin(\theta + \alpha) = -1$
Least $f(\theta) = \frac{2}{35} = \frac{2}{35}$, when $\sin(\theta + \alpha) = 1$
$\sin(\theta + \alpha) = -1$: $\theta + 61.9° = 270°$, so $\theta = 208.1°$
Answer: (a) $17\sin(\theta + 61.9°)$ (b) greatest 2, least $\frac{2}{35}$, $\theta = 208.1°$

9 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.
Given the second is a H, P(first was a H) = $\frac{2}{3}$. Find x.

P(H then H) = $0.4x$, P(second H) = $0.4x + 0.6(1 - x)$
P(first H | second H) = $\frac{0.4x}{0.4x + 0.6(1 - x)} = \frac{2}{3}$
$0.4x = \frac{2}{3}(0.4x + 0.6 - 0.6x)$
Solve the linear equation for x: $x = \frac{3}{4}$
Answer: $x = \frac{3}{4}$

10 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 9 and its common difference d is non-zero. The 2nd, 4th and 7th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: $9 + d$, $9 + 3d$, $9 + 6d$.

Geometric: $(9 + 3d)^2 = (9 + d)(9 + 6d)$.

Expand: $9d^2 + 54d + 81 = 6d^2 + 63d + 81$, so $d(3d - 9) = 0$.

$d \neq 0$, so $d = 3$. Ratio $= \frac{18}{12} = \frac{3}{2}$.

Answer: $d = 3$, ratio $= \frac{3}{2}$