Day 217 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 16 and upper quartile 31. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 8 observations have mean 19. The largest is 60, an outlier, and it is removed. Find the new mean.

IQR = 31 − 16 = 15, so 1.5 × IQR = 22.5

Lower limit = 16 − 22.5 = −6.5, upper limit = 31 + 22.5 = 53.5

$\Sigma x = 8 \times 19 = 152$, so without 60: $\Sigma x = 92$

New mean = $\frac{92}{7}$

Answer: (a) −6.5 to 53.5 (b) 13.14

2 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 29.4 m s^{-1} from the top of a vertical cliff of height 122.5 m, and lands on horizontal ground.

Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.

Vertically (up positive): $-122.5 = -4.9t^2$, so $t^2 = 25$

t = 5 s (the positive root)

Horizontal distance = 29.4 × 5 = 147 m

On landing: $v_x = 29.4$, $v_y = 0 - 9.8 \times 5 = -49$

speed = $\sqrt{(29.4^2 + 49^2)} = 57.1$, tan angle = $\frac{49}{29.4}$, angle = 59.0° below the horizontal

Answer: (a) 5 s, 147 m (b) 57.1 m s^{-1} at 59.0° below the horizontal

3 Integrals of the form f'(x)/f(x) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$$\int_0^2 \frac{x}{x^2 + 3}\,dx$$

Numerator is $\frac{1}{2}$ × derivative of the denominator

Integral = $[\frac{1}{2}\ln(x^2 + 3)]$

= $\frac{1}{2}(\ln 7 - \ln 3)$

Answer: $-\frac{1}{2}\ln 3 + \frac{1}{2}\ln 7$

4 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3 + 3\sqrt{6}}{5 - 2\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(5 + 2\sqrt{6})$.

Numerator: $(3 + 3\sqrt{6})(5 + 2\sqrt{6}) = 51 + 21\sqrt{6}$.

Denominator: $5^2 - 2^2(6) = 1$.

So the fraction = $\frac{51 + 21\sqrt{6}}{1}$.

Answer: $51 + 21\sqrt{6}$

5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - t$, $y = t^3 + 4t$.
Find the equation of the tangent at the point where $t = 2$,
in the form $y = mx + c$.

$dx/dt = 6t - 1$, $dy/dt = 3t^2 + 4$

At $t = 2$: $dy/dx = \frac{16}{11} = \frac{16}{11}$; point (10, 16)

$y - 16 = \frac{16}{11}(x - 10)$

Answer: $\mathbf{y = \frac{16}{11}x + \frac{16}{11}}$

6 One-tailed binomial test with data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school says that 25% of its pupils walk to school. 20 pupils are surveyed, and 6 walk.
Test, at the 1% significance level, the claim that the proportion is higher.
Give your conclusion in context.

H_0: $p = 0.25$, H_1: $p > 0.25$. Let $X \sim B(20, 0.25)$

$P(X \geq 6) = 1 - P(X \leq 5) = 0.383$

0.383 is greater than 1%, so do not reject H_0

There is insufficient evidence that the proportion is higher than the claimed 0.25

Answer: Do not reject H_0: p-value 0.383 > 1%

7 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{x^3 - 8x^2 + 19x - 6}{(x-3)(x-4)}$ in the form $ax + b + \frac{C}{(x-3)} + \frac{D}{(x-4)}$.

Divide: the numerator has degree 3, the denominator degree 2.

$(x^3 - 8x^2 + 19x - 6) \div (x^2 - 7x + 12)$ gives quotient $x - 1$.

Remainder: 6.

Put $x = 3$: remainder $= C(-1)$ so $C = -6$. Put $x = 4$: $D = 6$.

Answer: $\mathbf{x - 1 - \frac{6}{(x-3)} + \frac{6}{(x-4)}}$

8 Displacement of the form R sin(ωt + φ) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a line. Its displacement from a fixed point O at time t seconds is s metres, where $s = 6 \sin 3t + 8 \cos 3t$.
Find (a) the greatest distance of P from O, (b) the greatest speed of P, (c) the first time P is at its greatest distance from O (3 s.f.).

Write $s = R \sin(3t + \varphi)$ with $R \cos \varphi = 6$ and $R \sin \varphi = 8$

$R = \sqrt{(6^2 + 8^2)} = 10$ and $\tan \varphi = \frac{8}{6}$, so $\varphi = 0.9273$ rad

Greatest distance = R = 10

$v = \frac{ds}{dt} = 10 \times 3 \cos(3t + \varphi)$, so greatest speed = 30

s is greatest when $3t + \varphi = \frac{\pi}{2}$: $t = (1.5708 - 0.9273) \div 3$

Answer: (a) 10 m (b) 30 m $\mathbf{s^{-1}}$ (c) 0.215 s

9 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + s\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $3s = -6 + 3t$; $2 + s = 4 - 3t$; $-3 - s = 1 - 3t$

Solving the first two: $s = -1$, $t = 1$; the third equation is satisfied

Intersection: $(-3, 1, -2)$

$\cos\varphi = \frac{|9|}{\sqrt{11}\sqrt{27}} = 0.5222$, $\varphi = 58.5°$

Answer: Lines meet at $(-3, 1, -2)$; angle 58.5°

10 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.

Line l_2 is perpendicular to l_1 at B and meets the

x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{3-2}{-2+5} = \frac{1}{3}$.

Gradient of $l_2 = -3$. Equation: $y - 3 = -3(x + 2)$.

$y = 0$: $x = -1$, so C$(-1, 0)$.

AB $= \sqrt{10}$, BC $= \sqrt{10}$: area = ½ × AB × BC.

Answer: C$(-1, 0)$; area 5