Day 215 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 10 and upper quartile 22. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 15 observations have mean 12. The largest is 52, an outlier, and it is removed. Find the new mean.

IQR = 22 − 10 = 12, so 1.5 × IQR = 18
Lower limit = 10 − 18 = −8, upper limit = 22 + 18 = 40
Σx = 15 × 12 = 180, so without 52: Σx = 128
New mean = $\frac{128}{14}$

Answer: (a) −8 to 40 (b) 9.143

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_3 x + \log_3 y = 4$
$x - y = 24$

$\log_3(xy) = 4$, so $xy = 3^4 = 81$
$x = y + 24$: $y^2 + 24y - 81 = 0$
$(y - 3)(y + 27) = 0$; $y > 0$ so $y = 3$
Then $x = y + 24$
Answer: x = 27, y = 3

3 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 6 = 0$ differ by 1.
Find the possible values of k.

Let the roots be α and α + 1.
Product: $\alpha(\alpha + 1) = 6$, so $\alpha^2 + 1\alpha - 6 = 0$.
$(\alpha - 2)(\alpha + 3) = 0$, so $\alpha = 2$ or $\alpha = -3$.
Sum: $2\alpha + 1 = -k$, so $k = -(2\alpha + 1) = -5$ or 5.
Answer: k = ±5

4 Trigonometric integrals with identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use a suitable trigonometric identity to find the exact value of

$$\int_0^{\pi/6} \cos^4 3x \, dx$$

Use $\sin^2 A$ or $\cos^2 A$ twice: write as $(\ldots)^2$ with the double angle formula
Integrate term by term
Substitute the limits
Answer: $\frac{1}{16}\pi$

5 Proof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sam claims that $n^2 + 3n + 43$ is prime for every positive integer n.
Show that Sam is wrong by finding the smallest value of n that is a counter-example.

Test n = 1, 2, 3, ... in turn until the value is not prime.
For n = 39: $39^2 + 339 + 43 = 1681$.
1681 = 41 × 41, which is not prime.
All smaller values of n give a prime, so n = 39 is the smallest.
Answer: n = 39 (value 1681 = 41 × 41)

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6 Particle on a smooth slope joined to hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (2 kg) and B (7 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a smooth slope, $\sin\alpha = \frac{4}{5}$, and B is 2 m above the ground.

Released from rest, find the further distance A moves up the slope after B lands (A does not reach the pulley).

A: $T - 2 \times 9.8 \times \frac{4}{5} = 2a$. B: $7 \times 9.8 - T = 7a$

Adding: $9.8(7 - 2 \times \frac{4}{5}) = 9a$, so $a = 5.88$

$T = 7(9.8 - 5.88) = 27.44$

When B lands: $v^2 = 2 \times 5.88 \times 2 = 23.52$

Then A decelerates at $g\sin\alpha = 7.84$: further distance $= \frac{23.52}{2 \times 7.84} = 1.5$

Answer: 1.5 m

7 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = \frac{2}{x^2 + 2}$ at the point where $x = 1$

meets the axes at A and B. Find the exact area of triangle OAB, where O is the origin.

At $x = 1$: $y = \frac{2}{3}$. Differentiate: gradient $= -\frac{4}{9}$

Tangent: $y - \frac{2}{3} = (-\frac{4}{9})(x - 1)$, so $y = -\frac{4}{9}x + \frac{10}{9}$

y-intercept $\frac{10}{9}$; x-intercept $\frac{5}{2}$

Area $= \frac{1}{2} \times |\frac{10}{9}| \times |\frac{5}{2}|$

Answer: $\frac{25}{18}$

8 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + ax^2 + bx - 15$, where a and b are constants.
$(x + 3)$ is a factor of $f(x)$, and $f(x)$ leaves remainder -35 when divided by $(x - 4)$.
Find a and b, and hence solve $f(x) = 0$.

$f(-3) = 0$: $-27 + 9a - 3b - 15 = 0$, so $9a - 3b - 42 = 0$.

$f(4) = -35$: $16a + 4b + 49 = -35$.

Solving: $a = -1$, $b = -17$.

$f(x) = x^3 - x^2 - 17x - 15 = (x + 3)(x + 1)(x - 5)$

Answer: $a = -1$, $b = -17$; $x = -3, -1, 5$

9 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A battery's life, in hours, is $N(77, 5^2)$.
Given that it exceeds 82, find the probability that it exceeds 91.

Required $= P(X > 91 \mid X > 82) = \frac{P(X > 91)}{P(X > 82)}$

$P(X > 91) = P(Z > 2.800) = 0.002555$

$P(X > 82) = P(Z > 1.000) = 0.1587$

$= \frac{0.002555}{0.1587} = 0.0161$

Answer: 0.0161

10 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 m, 120 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{5}{12}$, where α is the angle between string and rod. A 30 N particle hangs at P, AP = 1 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Moments about A: $T\sin\alpha \times 4 = 120 \times 2 + 30 \times 1$

$T \times \frac{5}{13} \times 4 = 270$, so $T = 175.5$

Horizontal: $H = T\cos\alpha = 162$. Vertical: $V + T\sin\alpha = 120 + 30$, so $V = 82.5$

Force $= \sqrt{(162^2 + 82.5^2)} = 182$, angle $= \tan^{-1}\left(\frac{82.5}{162}\right) = 27.0°$ above the horizontal

Answer: (a) 175.5 N (b) 182 N at 27.0° above the horizontal (away from the wall)