Day 214 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^3 - 12x^2 + 18x - 8 < 0$.

Use the factor theorem to find roots: 1, 4.

$f(x) = 2(x - 1)^2(x - 4)$.

Sign of f in each region (left to right): −, −, +.

Take the regions where $f < 0$ (include roots if ≤ or ≥).

Answer: $x < 1$ or $1 < x < 4$

2 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (5 kg) lies on a rough table with $\mu = 0.4$, and is joined by a light string over a smooth pulley (see diagram) to particle B (10 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.

Released from rest, the string is cut when B has moved 1.75 m. Find the further distance A slides before stopping.

A: $T - 0.4 \times 5 \times 9.8 = 5a$. B: $10 \times 9.8 \times \frac{4}{5} - T = 10a$

Adding: $9.8(10 \times \frac{4}{5} - 0.4 \times 5) = 15a$, so $a = 3.92$

$T = 5(3.92 + 0.4 \times 9.8) = 39.2$

When cut: $v^2 = 2 \times 3.92 \times 1.75 = 13.72$. Then A decelerates at $\mu g = 3.92$

Further distance $= \frac{13.72}{2 \times 3.92} = 1.75$

Answer: 1.75 m

3 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{3}{8}) + \tan^{-1}(\frac{1}{4}))$.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{3}{8} + \frac{1}{4}}{1 - \frac{3}{8} \times \frac{1}{4}} = \frac{\frac{5}{8}}{\frac{29}{32}}$$

Answer: $\frac{20}{29}$

4 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 recorded masses (grams) have mean 50 and standard deviation 4.

One value was recorded as 51 but should have been 63.

Find the correct mean and the correct standard deviation.

$\Sigma x = 12 \times 50 = 600$, so corrected $\Sigma x = 600 - 51 + 63 = 612$

Correct mean $= \frac{612}{12} = 51.00$

$\Sigma x^2 = 12(16 + 2500) = 30192$, corrected $\Sigma x^2 = 30192 - 2601 + 3969 = 31560$

Variance $= \frac{31560}{12} - 51.000^2 = 29.000$, $\sigma = 5.385$

Answer: mean 51.0, standard deviation 5.39

5 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $x^2 + y^2 + 4x - 8y - 149 = 0$
passes through P(−14, 9). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (−2, 4). Gradient of radius CP = $-\frac{5}{12}$.

Tangent gradient = $\frac{12}{5}$: $-12x + 5y = 213$.

x = 0: y = $\frac{213}{5}$. y = 0: x = $-\frac{71}{4}$.

Area = ½ × $\frac{71}{4}$ × $\frac{213}{5}$.

Answer: $\frac{15123}{40}$

6 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 15 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially θ = 90, and after 5 minutes θ = 60.
Find A and k, then find when θ = 47 (3 s.f.).

t = 0: 15 + A = 90 so A = 75

t = 5: $15 + 75e^{-5k} = 60$ so $e^{-5k} = \frac{3}{5}$

k = $\frac{1}{5}\ln\frac{5}{3} = 0.102$

$75\,e^{-kt} = 32$ so t = $\frac{1}{k}\ln\frac{75}{32}$

t = 8.34 minutes

Answer: A = 75, k = 0.102, t = 8.34 minutes

7 Range and greatest height to speed and angle (AQA, Edexcel, OCR, MEI, Eduqas)

A golf ball is hit from a point O on horizontal ground. It lands 48 m from O, and reaches a maximum height of 16 m above the ground.
Find the angle of projection and the speed of projection, to 3 s.f. Take g = 9.8 m s^{-2}.

Vertical: $0 = u_y^2 - 2 \times 9.8 \times 16$, so $u_y = 17.71$

Time of flight = $\frac{2u_y}{9.8}$ = 3.614 s, so $u_x = \frac{48}{3.614} = 13.28$

$\tan\theta = \frac{u_y}{u_x} = 1.333$, so θ = 53.1°

Speed = $\sqrt{(u_x^2 + u_y^2)} = 22.1$

Answer: θ = 53.1°, speed 22.1 m s^{-1}

8 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{4}{(x-1)} < \frac{5}{(x-4)}$.

Multiply by the positive quantity $(x-1)^2(x-4)^2$:
$4(x-1)(x-4)^2 < 5(x-4)(x-1)^2$.
Move to one side and factorise: $(x-1)(x-4)(-x-11) < 0$.
Critical values: 1, 4, −11. Test the four regions.
Answer: $-11 < x < 1$ or $x > 4$

9 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.

Find (a) the total number of runners, (b) an estimate of the number with $15 \leq t < 34$.

Frequency = frequency density × width. Bar 18 to 26: height 7 squares, width 8, frequency 112

So one square of frequency density = 2 runners per second

Class frequencies = 72, 48, 112, 24, 32

Total = 288

Estimate = $2 \times [4(3) + 7(8) + 2(6) + 4(2)] = 176$

Answer: (a) 288 (b) 176

10 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two drones P and Q move so that, t seconds after launch, their position vectors in metres are

$\mathbf{r}_P = \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} -7 \\ -12 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -11 \\ -11 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = 0$

$-56 + t(14) = 0$, so $t = 4$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix}$, with length $\sqrt{19}$

Answer: t = 4, minimum distance = $\sqrt{19}$ m