Day 205 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan(2\theta + 20°) = -1$
for $0° \le \theta \le 360°$.

Let $u = 2\theta + 20°$; then $\tan u = -1$
Base solutions: $u = -45°$ (+ multiples of 180°)
$\theta = (u - 20°) \div 2$, keeping those in range
Answer: θ = 57.5°, 147.5°, 237.5°, 327.5°

2 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1000 kg tows a trailer of mass 600 kg along a horizontal road, with driving force 1360 N. Resistances are 400 N on the car and 480 N on the trailer.

(a) Find the tension in the tow bar. At 6.4 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{1360 - 400 - 480}{1600} = 0.3$

Trailer: $T - 480 = 600 \times 0.3$, so $T = 660$

After the break the trailer decelerates at $\frac{480}{600} = 0.8$, and stops after $t = \frac{6.4}{0.8} = 8$ s

Trailer travels $\frac{6.4}{2} \times 8 = 25.6$ m

Car: $a = \frac{1360 - 400}{1000} = 0.96$, so $s = 6.4 \times 8 + ½ \times 0.96 \times 8^2 = 81.92$ m

Distance apart = $81.92 - 25.6$
Answer: (a) 660 N (b) 56.32 m

3 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 4x^2 - 16x + 21$.

Complete the square: $y = 4(x - 2)^2 + 5$.
Step 1: $y = x^2 \rightarrow y = 4x^2$: a stretch, scale factor 4, parallel to the y-axis.
Step 2: replace x by $(x - 2)$ and add 5: translation by $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.
Answer: a stretch, scale factor 4, parallel to the y-axis, then translation $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

4 Segments: exact area and perimeter (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB of a circle, centre O and radius 8 cm, subtends an angle of $\frac{\pi}{3}$ at O.

Find the exact area and the exact perimeter of the minor segment cut off by AB.

Sector OAB: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{3}$; triangle OAB: $\frac{1}{2}r^2 \sin\theta = 16\sqrt{3}$

Area = $\frac{32}{3}\pi - 16\sqrt{3}$

Chord AB = $2r \sin(\theta/2) = 8$

Arc AB = $r\theta = \frac{8}{3}\pi$

Perimeter = arc + chord
Answer: Area = $\frac{32}{3}\pi - 16\sqrt{3}$ cm^2, perimeter = $8 + \frac{8}{3}\pi$ cm

5 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 observations x, $\Sigma(x - 30) = 40$ and $\Sigma(x - 30)^2 = 380$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 30) = \frac{40}{20} = 2$, so mean of $x = 32$

Variance $= \frac{380}{20} - (2)^2 = 15$

$\sigma = \sqrt{15}$

$\Sigma x^2 = \Sigma(x - 30)^2 + 60\,\Sigma(x - 30) + 20 \times 30^2$

$= 380 + 2400 + 18000$

Answer: (a) 32 (b) 3.87 (c) 20780

6 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(-5\mathbf{i} + \mathbf{j})$ m with velocity $(14\mathbf{i} + 11\mathbf{j})$ m s^{-1}. At t = 8, P has velocity $(-18\mathbf{i} - 13\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i} + \mathbf{j})$, (c) the position vector of P at that time.

$a = \frac{v - u}{t} = \frac{(-18\mathbf{i} - 13\mathbf{j}) - (14\mathbf{i} + 11\mathbf{j})}{8} = (-4\mathbf{i} - 3\mathbf{j})$

$v = (14\mathbf{i} + 11\mathbf{j}) + t(-4\mathbf{i} - 3\mathbf{j})$

Parallel to $(\mathbf{i} + \mathbf{j})$: $(-4t + 14) \times 1 = (-3t + 11) \times 1$, giving $t = 3$

$r = (-5\mathbf{i} + \mathbf{j}) + 3(14\mathbf{i} + 11\mathbf{j}) + \frac{1}{2} \times 3^2\,(-4\mathbf{i} - 3\mathbf{j}) = (19\mathbf{i} + 20.5\mathbf{j})$

Answer: (a) $(-4\mathbf{i} - 3\mathbf{j})$ m s^{-2} (b) t = 3 (c) $(19\mathbf{i} + 20.5\mathbf{j})$ m

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x^3\sqrt{7 - x}$, for $0 < x < 7$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{2x^2(42 - 7x)}{2\sqrt{7 - x}}$

$\frac{dy}{dx} = 0$: $42 - 7x = 0$, so $x = 6$

$y = 2 \times 6^3 \times 1$

Gradient changes from positive to negative: a maximum

Answer: (6, 432), maximum

8 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -1$ and $y = -1$, so $p = 1$ and $q = -1$.

$y = \frac{a}{x + 1} - 1$.

P(−3, 1): $1 = \frac{a}{-2} - 1$, so $a = -4$.

Answer: $a = -4$, $p = 1$, $q = -1$

9 PMCC from summary data and a hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

For 10 pairs of values, $S_{xx} = 324$, $S_{yy} = 169$ and $S_{xy} = 179$.

(a) Calculate the product moment correlation coefficient r.

(b) The critical value for a two-tailed test at the 5% level with n = 10 is 0.6319. Carry out the test of H_0: $\rho = 0$ against H_1: $\rho \neq 0$.

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}} = \frac{179}{\sqrt{(324 \times 169)}} = \frac{179}{234} = 0.7650$$

H_0: $\rho = 0$, H_1: $\rho \neq 0$

$|r| = 0.7650 >$ the critical value 0.6319

Reject H_0: there is sufficient evidence of correlation between the variables

Answer: (a) r = 0.765 (b) reject H_0

10 Indices: simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously, giving integer x and y:

$8^x \times 16^y = 128$

$27^x \div 9^y = 3$

Write each side with prime bases: $128 = 2^7$, $3 = 3^1$.

First: $2^{3x} \times 2^{4y} = 2^7$, so $3x + 4y = 7$.

Second: $3^{3x - 2y} = 3^1$, so $3x - 2y = 1$.

Solving the pair of linear equations: x = 1, y = 1.

Answer: x = 1, y = 1