Day 202 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(36, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find $P(X > 57)$.

The area 0.8 leaves 0.200 in total in the tails, 0.100 in each

$P(X < 25) = 0.100$ so $\frac{25 - 36}{\sigma} = -1.2816$

$\sigma = \frac{11}{1.2816} = 8.583$

$P(X > 57) = P(Z > 2.447) = 0.00721$

Answer: (a) 8.58 (b) 0.00721

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_5 x - \log_5 y = 1$
$x + y = 42$

$\log_5(x/y) = 1$, so $x = 5^1 y = 5y$
$5y + y = 42$, so $6y = 42$
Answer: x = 35, y = 7

3 Screening test and Bayes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rare allele is present in 0.5% of a population. A test is positive for 98% of those with it and for 8% of those without it. Find P(positive) and P(has it | positive).

$P(\text{positive}) = 0.005 \times 0.98 + 0.995 \times 0.08$
$= 0.0049 + 0.0796 = 0.0845$

$P(\text{has it} \mid \text{positive}) = \frac{0.0049}{0.0845}$

$= 0.05799$

Answer: (a) 0.0845 (b) 0.0580

4 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-6 - 5\sqrt{3}}{6 + 3\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(6 - 3\sqrt{3})$.
Numerator: $(-6 - 5\sqrt{3})(6 - 3\sqrt{3}) = 9 - 12\sqrt{3}$.
Denominator: $6^2 - 3^2(3) = 9$.
So the fraction = $\frac{9 - 12\sqrt{3}}{9}$.

Answer: $1 - \frac{4}{3}\sqrt{3}$

5 Slide down a rough slope then along rough ground (CIE)

A particle starts from rest at the top of a rough slope of length 20 m ($\sin\alpha = \frac{4}{5}$, $\mu = 0.4$), then slides along rough horizontal ground with the same μ.
Use energy to find (a) its speed at the foot of the slope (3 s.f.), (b) the distance it slides on the ground. Take $g = 9.8$ m s^{-2}.

On the slope: $\frac{1}{2} m v^2 = m g \times 20 \sin\alpha - \mu m g \cos\alpha \times 20$

$v^2 = 2 \times 9.8 \times 20(\frac{4}{5} - 0.4 \times \frac{3}{5}) = 219.52$, so $v = 14.82$

On the ground: $\frac{1}{2} m v^2 = \mu m g x$, so $x = \frac{v^2}{2 \mu g} = \frac{219.52}{7.84} = 28$

Answer: (a) 14.8 m s^{-1} (b) 28 m

6 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + 3y^2 - 2y$.
Find dy/dx and d^2y/dx^2 when $y = 2$.

$dx/dy = 3y^2 + 6y - 2 = 22$ when $y = 2$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{22}$

$d^2x/dy^2 = 18$ when $y = 2$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{18}{22^3}$

Answer: $dy/dx = \frac{1}{22}$, $d^2y/dx^2 = -\frac{9}{5324}$

7 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = \sqrt{-x^2 + 2x + 24}$.
Find the largest possible domain of g, and the corresponding range.

Need $-x^2 + 2x + 24 \geq 0$, i.e. $(x + 4)(x - 6) \leq 0$.
Domain: $-4 \leq x \leq 6$.
Complete the square: $-x^2 + 2x + 24 = 25 - (x - 1)^2$.
Greatest value of $g = \sqrt{25} = 5$, and least value is 0.
Answer: $-4 \leq x \leq 6$; $0 \leq g(x) \leq 5$

8 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = -1$ and

$u_{n+1} = \frac{1}{1 - u_n}$.

Show that the sequence is periodic and find

$\sum_{r=1}^{80} u_r$.

$u_1 = -1$, $u_2 = \frac{1}{2}$, $u_3 = 2$, $u_4 = -1$.

$u_4 = u_1$, so the sequence repeats with period 3.

One period sums to $\frac{3}{2}$.

$80 = 3 \times 26 + 2$: 26 full periods plus the first 2 terms.

Answer: $\frac{77}{2}$

9 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{4}{16 + 12\sin\theta + 5\cos\theta}$

(a) Write $12\sin\theta + 5\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.
(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{12^2 + 5^2} = 13$, $\tan\alpha = \frac{5}{12}$, $\alpha = 22.6°$

Denominator $= 16 + 13\sin(\theta + 22.6°)$, which lies between 3 and 29

Greatest $f(\theta) = \frac{4}{3} = \frac{4}{3}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{4}{29} = \frac{4}{29}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 22.6° = 270°$, so $\theta = 247.4°$

Answer: (a) $13\sin(\theta + 22.6°)$ (b) greatest $\frac{4}{3}$, least $\frac{4}{29}$, $\theta = 247.4°$

10 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (3 m, 60 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{5}{12}$, where α is the angle between string and rod. A 50 N particle hangs at P, AP = 1.5 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Moments about A: $T\sin\alpha \times 3 = 60 \times 1.5 + 50 \times 1.5$

$T \times \frac{5}{13} \times 3 = 165$, so $T = 143$

Horizontal: $H = T\cos\alpha = 132$. Vertical: $V + T\sin\alpha = 60 + 50$, so $V = 55$

Force $= \sqrt{(132^2 + 55^2)} = 143$, angle $= \tan^{-1}(\frac{55}{132}) = 22.6°$ above the horizontal

Answer: (a) 143 N (b) 143 N at 22.6° above the horizontal (away from the wall)