Day 200 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of

$y = -2f(-\frac{1}{2}x - 2) + 1$.

Inside f: replace x by $-\frac{1}{2}x - 2$, so x-coordinates become $(x + 2) \div (-\frac{1}{2})$.

Outside: y-coordinates become $-2 \times y + 1$.

A: (8, 5) B: (4, −9)

C: (0, 9) D: (−16, 1)

Answer: A′(8, 5), B′(4, −9), C′(0, 9), D′(−16, 1)

2 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 13 and upper quartile 22. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 10 observations have mean 22. The largest is 41, an outlier, and it is removed. Find the new mean.

IQR = 22 − 13 = 9, so 1.5 × IQR = 13.5

Lower limit = 13 − 13.5 = −0.5, upper limit = 22 + 13.5 = 35.5

$\Sigma x = 10 \times 22 = 220$, so without 41: $\Sigma x = 179$

New mean = $\frac{179}{9}$

Answer: (a) −0.5 to 35.5 (b) 19.89

3 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_2(x + 3) + \log_2(x - 12) = 4$,

justifying clearly any root that you reject.

$(x + 3)(x - 12) = 2^4 = 16$

$x^2 - 9x - 52 = 0$

$(x - 13)(x + 4) = 0$, so $x = 13$ or $x = -4$

$x = -4$ makes $x + 3 = -1 < 0$ and log of a negative number is undefined

Answer: x = 13

4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(4r + 4) = 2808$, find n.

$\sum_{r=1}^{n}(4r + 4) = 4 \times \frac{n(n + 1)}{2} + 4n$.

$\frac{4\,n(n + 1)}{2} + 4n = 2808$, so $4n^2 + 12n - 5616 = 0$.

Solving (n > 0): n = 36.

Answer: n = 36

5 Identity then equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$.

(b) Hence solve $\cos^4\theta - \sin^4\theta = -\frac{1}{3}$ for $0 \leq \theta < 360°$ (1 d.p.).

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \cos 2\theta \times 1$

$\cos 2\theta = -\frac{1}{3}$, so $2\theta = 109.5°, 250.5°, 469.5°, 610.5°$ $(0 \leq 2\theta < 720°)$

Halve each value

Answer: θ = 54.7°, 125.3°, 234.7°, 305.3°

6 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.
Find (a) the total number of runners, (b) an estimate of the number with $16 \leq t < 48$.

Frequency = frequency density × width. Bar 12 to 18: height 1 squares, width 6, frequency 12
So one square of frequency density = 2 runners per second
Class frequencies = 168, 12, 120, 112, 120
Total = 532
Estimate = 2 × [1(2) + 5(12) + 7(8) + 5(10)] = 336
Answer: (a) 532 (b) 336

7 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 8 m and mass 30 kg rests on supports at C and D, where AC = 3 m and DB = 3 m. A girl of mass 50 kg stands on it at distance x m from A.
Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs $R_C \geq 0$ and $R_D \geq 0$. Limiting cases: $R_C = 0$ and $R_D = 0$ in turn

Tilting about D (girl beyond D): $30(1) = 50(x - 5)$, so $x = 5 + \frac{30 \times 1}{50} = 5.6$

Tilting about C (girl before C): $30(1) = 50(3 - x)$, so $x = 3 - \frac{30 \times 1}{50} = 2.4$

Answer: $2.4 \leq x \leq 5.6$

8 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = t$, $y = 4t^2$. The line $y = -8x + 32$
meets C at A and B. Find the exact length AB.

Substitute: $4t^2 = -8t + 32$.
$4t^2 + 8t - 32 = 0 \Rightarrow t = -4$ or $t = 2$.
A(−4, 64), B(2, 16).
$AB^2 = (-6)^2 + 48^2 = 2340$.
Answer: $AB = 6\sqrt{65}$

9 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(5\mathbf{i} - 5\mathbf{j})$ N, $(2\mathbf{i} + k\mathbf{j})$ N and $(-2\mathbf{i} + 8\mathbf{j})$ N act on a particle of mass 2 kg. The resultant is parallel to $(5\mathbf{i} + 9\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(5)\mathbf{i} + (3 + k)\mathbf{j}$

Parallel to $(5\mathbf{i} + 9\mathbf{j})$: $\frac{3 + k}{5} = \frac{9}{5}$, so $k = 6$

$R = (5\mathbf{i} + 9\mathbf{j})$ N, so $|R| = \sqrt{(5^2 + 9^2)} = 10.30$
$a = |R| \div 2$
Answer: (a) $k = 6$ (b) 5.15 m s^{-2}

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} -7 \\ -4 \\ -7 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ -4 \end{pmatrix} + t\begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $-7 + s = -4$; $-4 + 3s = -1 - 3t$; $-7 + 3s = -4 - 3t$
Solving the first two: $s = 3$, $t = -2$; the third equation is satisfied
Intersection: (−4, 5, 2)
$\cos\varphi = \frac{|-18|}{\sqrt{19}\sqrt{18}} = 0.9733$, $\varphi = 13.3°$
Answer: Lines meet at (−4, 5, 2); angle 13.3°