Day 20 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass M kg, carrying a passenger of mass 80 kg, hangs from a vertical cable. The tension in the cable is 5512 N and the floor exerts a force of 832 N on the passenger.

Find (a) the acceleration of the lift, with its direction, (b) the value of M. Take g = 9.8 m s^{-2}.

Passenger (upwards positive): $832 - 80 \times 9.8 = 80a$, so $a = 0.6$

$a = 0.6$ m s^{-2}, so the acceleration is upwards

Lift and passenger: $5512 - (M + 80) \times 9.8 = (M + 80) \times 0.6$

$(M + 80)(9.8 + 0.6) = 5512$, so $M + 80 = \frac{5512}{10.4} = 530$

Answer: (a) 0.6 m s^{-2} upwards (b) M = 450

2 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

15 recorded lengths (cm) have mean 49 and standard deviation 6.

One value was recorded as 65 but should have been 54.

Find the correct mean and the correct standard deviation.

$\Sigma x = 15 \times 49 = 735$, so corrected $\Sigma x = 735 - 65 + 54 = 724$

Correct mean $= \frac{724}{15} = 48.27$

$\Sigma x^2 = 15(36 + 2401) = 36555$, corrected $\Sigma x^2 = 36555 - 4225 + 2916 = 35246$

Variance $= \frac{35246}{15} - 48.267^2 = 20.062$, $\sigma = 4.479$

Answer: mean 48.3, standard deviation 4.48

3 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_2(x + 11) + \log_2(x - 1) = 6$,

justifying clearly any root that you reject.

$(x + 11)(x - 1) = 2^6 = 64$

$x^2 + 10x - 75 = 0$

$(x - 5)(x + 15) = 0$, so $x = 5$ or $x = -15$

$x = -15$ makes $x + 11 = -4 < 0$ and log of a negative number is undefined

Answer: x = 5

4 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{x + 7} = x + 1$.

Square both sides: $x + 7 = (x + 1)^2$.

$x^2 + x - 6 = 0$, so $x = -3$ or $x = 2$.

Check in $\sqrt{x + 7} = x + 1$: $x = 2$ gives $3 \geq 0$ (valid).

$x = -3$ gives right-hand side $-2 < 0$, so it is rejected.

Answer: x = 2

5 Segments: exact area and perimeter (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB of a circle, centre O and radius 5 cm, subtends an angle of $\frac{2\pi}{3}$ at O.

Find the exact area and the exact perimeter of the minor segment cut off by AB.

Sector OAB: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times \frac{2\pi}{3}$; triangle OAB: $\frac{1}{2}r^2\sin\theta = \frac{25}{4}\sqrt{3}$

Area $= \frac{25}{3}\pi - \frac{25}{4}\sqrt{3}$

Chord AB $= 2r\sin(\theta/2) = 5\sqrt{3}$

Arc AB $= r\theta = \frac{10}{3}\pi$

Perimeter = arc + chord

Answer: Area $= \frac{25}{3}\pi - \frac{25}{4}\sqrt{3}$ cm^2, perimeter $= \frac{10}{3}\pi + 5\sqrt{3}$ cm

6 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2-\sqrt{6}}{2-\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(2 + \sqrt{6})$.

Numerator: $(-2 - \sqrt{6})(2 + \sqrt{6}) = -10 - 4\sqrt{6}$.

Denominator: $2^2 - 1^2(6) = -2$.

So the fraction = $\frac{-10-4\sqrt{6}}{-2}$.

Answer: $5 + 2\sqrt{6}$

7 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 300 kg along a horizontal road, with driving force 1310 N. Resistances are 800 N on the car and 150 N on the trailer.

(a) Find the tension in the tow bar. At 5 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{1310 - 800 - 150}{1800} = 0.2$

Trailer: $T - 150 = 300 \times 0.2$, so $T = 210$

After the break the trailer decelerates at $\frac{150}{300} = 0.5$, and stops after $t = \frac{5}{0.5} = 10$ s

Trailer travels $\frac{5}{2} \times 10 = 25$ m

Car: $a = \frac{1310 - 800}{1500} = 0.34$, so $s = 5 \times 10 + ½ \times 0.34 \times 10^2 = 67$ m

Distance apart = $67 - 25$

Answer: (a) 210 N (b) 42 m

8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{4}{35 + 20\sin\theta + 21\cos\theta}$

(a) Write $20\sin\theta + 21\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{20^2 + 21^2} = 29$, $\tan\alpha = \frac{21}{20}$, $\alpha = 46.4°$

Denominator = $35 + 29\sin(\theta + 46.4°)$, which lies between 6 and 64

Greatest $f(\theta) = \frac{4}{6} = \frac{2}{3}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{4}{64} = \frac{1}{16}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 46.4° = 270°$, so $\theta = 223.6°$

Answer: (a) $29\sin(\theta + 46.4°)$ (b) greatest $\frac{2}{3}$, least $\frac{1}{16}$, $\theta = 223.6°$

9 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has its centre in the first quadrant and touches both the x-axis and the y-axis. It passes through the point (1, 2). Find the two possible equations of the circle.

Centre (r, r), radius r: $(x - r)^2 + (y - r)^2 = r^2$.

Substitute (1, 2): $(1 - r)^2 + (2 - r)^2 = r^2$.

$r^2 - 6r + 5 = 0$.

$(r - 1)(r - 5) = 0$, so $r = 1$ or $r = 5$.

Answer: $(x - 1)^2 + (y - 1)^2 = 1$ or $(x - 5)^2 + (y - 5)^2 = 25$

10 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: p = 0.4 against H_1: p > 0.4 uses a sample of 30 at the 5% level.

(a) Find the critical region for X, the number of successes.

(b) Find the actual significance level.

(c) Find P(Type II error) when in fact p = 0.5.

X ~ B(30, 0.4). P(X ≥ 17) = 0.0481 and P(X ≥ 16) = 0.0971

The first is at most 0.05 and the second is not, so the critical region is X ≥ 17

Actual significance level = P(X ≥ 17) = 0.0481

Type II error: H_0 is not rejected, so X ≤ 16, with X ~ B(30, 0.5)

P(X ≤ 16) = 0.708

Answer: (a) X ≥ 17 (b) 0.0481 (c) 0.708