Day 199 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 637
inclusive that are divisible by 6 or by 10, but not by both.

Multiples of 6: 106 terms (6 to 636): sum = 34026.
Multiples of 10: 63 terms: sum = 20160.
Multiples of both, i.e. of 30: 21 terms: sum = 6930.
Required sum = 34026 + 20160 − 2 × 6930.
Answer: 40326

2 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A radioactive sample has mass $m = m_0 e^{-kt}$ grams, t in days.
Initially m = 160 and after 3 days m = 128.
Find k, and the time for the mass to fall to 16 grams, to 3 s.f.

$m_0 = 160$. $160\, e^{-3k} = 128$ so $e^{-3k} = \frac{4}{5}$

$k = \frac{1}{3} \ln \frac{5}{4} = 0.0744$

$160\, e^{-kt} = 16$ so $e^{kt} = 10$, $t = \frac{1}{k} \ln 10$

t = 31.0 days
Answer: k = 0.0744, t = 31.0 days

3 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-4\mathbf{i} + 4\mathbf{j})$ N, $(-2\mathbf{i} + k\mathbf{j})$ N and $(-\mathbf{i} + 6\mathbf{j})$ N act on a particle of mass 4 kg. The resultant is parallel to $(-7\mathbf{i} + 13\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(-7)\mathbf{i} + (10 + k)\mathbf{j}$

Parallel to $(-7\mathbf{i} + 13\mathbf{j})$: $\frac{10 + k}{-7} = \frac{13}{-7}$, so k = 3

R = $(-7\mathbf{i} + 13\mathbf{j})$ N, so $|R| = \sqrt{(-7^2 + 13^2)} = 14.76$
a = |R| ÷ 4
Answer: (a) k = 3 (b) 3.69 m s^{-2}

4 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_3(x + 57) + \log_3(x - 23) = 4$,
justifying clearly any root that you reject.

$(x + 57)(x - 23) = 3^4 = 81$
$x^2 + 34x - 1392 = 0$
$(x - 24)(x + 58) = 0$, so x = 24 or x = −58
x = −58 makes x + 57 = −1 < 0 and log of a negative number is undefined
Answer: x = 24

5 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^3 + 3x^2 - 6x - 8 \leq 0$.

Use the factor theorem to find roots: −4, −1, 2.
f(x) = (x + 4)(x + 1)(x − 2).
Sign of f in each region (left to right): −, +, −, +.
Take the regions where f ≤ 0 (include roots if ≤ or ≥).
Answer: $x \leq -4$ or $-1 \leq x \leq 2$

6 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2\sqrt{5 - x}$, for $0 < x < 5$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{x(20 - 5x)}{2\sqrt{5 - x}}$

$\frac{dy}{dx} = 0$: $20 - 5x = 0$, so $x = 4$

$y = 4^2 \times 1$

Gradient changes from positive to negative: a maximum

Answer: (4, 16), maximum

7 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(80, 0.4)$.

(a) State why a normal approximation is suitable for X.

(b) Use a normal approximation to estimate $P(28 \leq X \leq 36)$, to 3 significant figures.

$np = 32$ and $nq = 48$, both greater than 5 (n large, p not extreme)

$Y \sim N(32, 19.2)$ with $\sigma = 4.382$

Apply the continuity correction:

$P(28 \leq X \leq 36) \approx P(27.5 < Y < 36.5) = 0.8478 - 0.1522 = 0.696$

Answer: 0.696

8 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 4 per hour. After a modification, 10 faults are found in 1 hour. Test at the 5% level whether the rate of faults has increased.

H_0: $\lambda = 4$ per hour, H_1: $\lambda > 4$ per hour

Over 1 hour: $X \sim Po(4)$

$P(X \geq 10) = 1 - P(X \leq 9) = 0.00813$

$0.00813 < 5\%$, so reject H_0

There is sufficient evidence that the rate has increased

Answer: Reject H_0 (p-value 0.00813)

9 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.

Line l_2 is perpendicular to l_1 at B and meets the x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{6}{2 + 7} = \frac{2}{3}$.

Gradient of $l_2 = -\frac{3}{2}$. Equation: $y - 6 = -\frac{3}{2}(x - 2)$.

$y = 0$: $x = 6$, so C(6, 0).

$AB = 3\sqrt{13}$, $BC = 2\sqrt{13}$: area = ½ × AB × BC.

Answer: C(6, 0); area 39

10 Car on a hill: power and maximum speed (CIE)

A car of mass 800 kg, with engine power 10.704 kW, climbs a straight road with $\sin\alpha = \frac{1}{25}$ against a constant resistance of 400 N.

Find (a) the greatest steady speed up the hill, (b) the acceleration at 5 m s^{-1}. Take g = 9.8 m s^{-2}.

Steady speed: driving force = $400 + 800 \times 9.8 \times \frac{1}{25} = 713.6$ N

$P = F\,v$, so $v = \frac{10704}{713.6} = 15$

At 5 m s^{-1}: $F = \frac{10704}{5} = 2140.8$ N

$F - 713.6 = 800\,a$, so $a = \frac{1427.2}{800}$

Answer: (a) 15 m s^{-1} (b) 1.78 m s^{-2}