**Day 197** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that $\frac{1-\cos 3\theta}{\theta \tan 4\theta}$ approaches as $\theta \to 0$.

$1 - \cos 3\theta \approx \frac{1}{2}(3\theta)^2 = \frac{9}{2}\theta^2$, $\tan 4\theta \approx 4\theta$

Expression $\approx \frac{\frac{9}{2}\theta^2}{\theta \times 4\theta} = \frac{9}{8}$

**Answer: $\frac{9}{8}$**

## 2 Least force to drag a box (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 25 N is pulled at constant speed along rough horizontal ground ($\mu = \frac{7}{24}$) by a force P N at angle θ above the horizontal.

(a) Find P when $\tan\theta = \frac{4}{3}$. (b) Find the least possible value of P as θ varies, and the corresponding θ to 1 d.p.

Vertical: $R + P\sin\theta = 25$. Horizontal: $P\cos\theta = \mu R = \frac{7}{24}(25 - P\sin\theta)$

$P(\cos\theta + \frac{7}{24}\sin\theta) = \frac{7}{24} \times 25$, so $P = \frac{175}{24} \div (\cos\theta + \frac{7}{24}\sin\theta)$

(a) $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$: $P = \frac{175}{24} \div \frac{5}{6} = 8.75$

(b) P is least when $\cos\theta + \frac{7}{24}\sin\theta$ is greatest. Its greatest value is $\sqrt{(1 + \frac{7}{24}^2)} = \frac{25}{24}$,

which occurs when $\tan\theta = \frac{7}{24}$. Then $P = \frac{175}{24} \div \frac{25}{24} = 7$ and $\theta = 16.3°$

**Answer: (a) 8.75 N (b) 7 N at θ = 16.3°**

## 3 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ab^x$ passes through (0, 3) and (4, 48).
Find the exact values of a and b, and the value of y when x = 6.

$ab^0 = 3$ and $ab^4 = 48$

Divide: $b^4 = 16$ so $b = 2$

$a = 3 \div 2^0 = 3$

$y = 3 \times 2^6$

**Answer: a = 3, b = 2, y = 192**

## 4 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^3 + ax^2 + bx - 36$, where a and b are constants.
(x + 2) is a factor of f(x), and f(x) leaves remainder 0 when divided by (x + 3).
Find a and b, and hence solve f(x) = 0.

$f(-2) = 0$: $-24 + 4a - 2b - 36 = 0$, so $4a - 2b - 60 = 0$.

$f(-3) = 0$: $9a - 3b - 117 = 0$.

Solving: $a = 9$, $b = -12$.

$f(x) = 3x^3 + 9x^2 - 12x - 36 = 3(x + 3)(x + 2)(x - 2)$

**Answer: a = 9, b = −12; x = −3, −2, 2**

## 5 Constant speed up a slope, force at an angle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 95 N is pulled at constant speed up a line of greatest slope of a rough plane ($\tan\alpha = \frac{3}{4}$, $\mu = \frac{7}{24}$) by a force P N at angle $\beta$ above the plane.

(a) Find P when $\tan\beta = \frac{7}{24}$. (b) Find the least possible value of P as $\beta$ varies.

Perpendicular to the plane: $R + P\sin\beta = 95\cos\alpha$, so $R = 95 \times \frac{4}{5} - P\sin\beta$

Along the plane: $P\cos\beta = 95\sin\alpha + \mu R$, so $P(\cos\beta + \mu\sin\beta) = 95(\sin\alpha + \mu\cos\alpha)$

(a) $P(\frac{24}{25} + \frac{7}{24} \times \frac{7}{25}) = 95(\frac{3}{5} + \frac{7}{24} \times \frac{4}{5})$, so $P = 76$

(b) P is least when $\cos\beta + \mu\sin\beta$ is greatest, i.e. $\sqrt{(1+\mu^2)} = \frac{25}{24}$ (when $\tan\beta = \frac{7}{24}$)

$P_{min} = 95(\frac{3}{5} + \frac{7}{24} \times \frac{4}{5}) \div \frac{25}{24}$

**Answer: (a) 76 N (b) 76 N**

## 6 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{6x^2 - 45x + 56}{(x+3)(x-4)^2}$ in the form

$\frac{A}{(x+3)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}$.

$6x^2 - 45x + 56 \equiv A(x-4)^2 + B(x+3)(x-4) + C(x+3)$.

Put $x = -3$: $245 = A(-7)^2$, so $A = 5$.

Put $x = 4$: $-28 = C(7)$, so $C = -4$.

Compare $x^2$: $6 = 1A + 1B$, so $B = 1$.

**Answer: A = 5, B = 1, C = −4**

## 7 Most likely value of a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(11, \frac{2}{5})$.

(a) Show that $P(X = r+1) \div P(X = r) = \frac{11-r}{r+1} \times \frac{2}{3}$.

(b) Find the most likely value of X and its probability (3 s.f.).

$P(X = r+1) \div P(X = r) = \frac{C(11, r+1)}{C(11, r)} \times \frac{p}{q} = \frac{11-r}{r+1} \times \frac{p}{q}$

With $p = \frac{2}{5}$ and $q = \frac{3}{5}$, $p \div q = \frac{2}{3}$

The ratio exceeds 1 when $(11-r)\frac{2}{5} > (r+1)\frac{3}{5}$, that is $r < 3.8$

So probabilities rise up to r = 4 and then fall; the mode is 4

$P(X = 4) = 0.236$

**Answer: 4 with probability 0.236**

## 8 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the point of inflection of

$y = (x+1)e^{-2x}$,

justifying that it is a point of inflection.

$dy/dx = e^{-2x}(1 - 2(x+1))$

$d^2y/dx^2 = 2e^{-2x}(2(x+1) - 2) = 0$, so $x = 0$

$d^2y/dx^2$ changes sign either side of $x = 0$

$y = 1$

**Answer: (0, 1)**

**Day 197** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 9 Pdf of the form a + bx with a given mean (CIE)

The pdf of X is f(x) = a + bx for $0 \leq x \leq 4$, and 0 otherwise, where a and b are constants. $E(X) = \frac{37}{15}$.

(a) Find a and b.
(b) Find P(X < 2).

Total area = 1: $4a + 8b = 1$

$E(X) = \int x(a + bx)\,dx = 8a + \frac{64}{3}b = \frac{37}{15}$

Solving the simultaneous equations: $a = \frac{3}{40}$, $b = \frac{7}{80}$

$P(X < 2) = \int f(x)\,dx$ from 0 to 2 $= \frac{3}{40} \times 2 + \frac{7}{80} \times 2$

$= \frac{13}{40}$

**Answer: (a) $a = \frac{3}{40}$, $b = \frac{7}{80}$ (b) $\frac{13}{40}$**

## 10 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = -2 + 5\cos t$, $y = -3 + 5\sin t$.
Find its Cartesian equation.

$\cos t = \frac{x + 2}{5}$, $\sin t = \frac{y + 3}{5}$.

Use $\cos^2 t + \sin^2 t = 1$:

$(x + 2)^2 + (y + 3)^2 = 25$.

**Answer: $(x + 2)^2 + (y + 3)^2 = 25$**