**Day 191** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 12 and upper quartile 21. An outlier is a value more than 2 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 8 observations have mean 20. The largest is 49, an outlier, and it is removed. Find the new mean.

IQR = 21 − 12 = 9, so 2 × IQR = 18

Lower limit = 12 − 18 = −6, upper limit = 21 + 18 = 39

$\Sigma x = 8 \times 20 = 160$, so without 49: $\Sigma x = 111$

New mean = $\frac{111}{7}$

**Answer: (a) −6 to 39 (b) 15.86**

## 2 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(12, 8) and B(6, 10). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - 12)^2 + 64 = (c - 6)^2 + 100$.

Expand and solve: $-12c = -72$, so c = 6.

Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$

$= \frac{1}{2}|6 \times 10 - 0 \times 8| = 30$.

**Answer: C(6, 0); area 30**

## 3 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_2(x + 3) + \log_2(x - 1) = 5$,

justifying clearly any root that you reject.

$(x + 3)(x - 1) = 2^5 = 32$

$x^2 + 2x - 35 = 0$

$(x - 5)(x + 7) = 0$, so x = 5 or x = −7

x = −7 makes x + 3 = −4 < 0 and log of a negative number is undefined

**Answer: x = 5**

## 4 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a apple is $N(88, 5^2)$ g. A apple is 'heavy' if it exceeds 91.

(a) Find the probability that a apple is heavy.

(b) 18 apples are chosen at random. Find the probability that at least 3 are heavy.

(a) P(X > 91) = P(Z > 0.600) = 0.2743

(b) Let Y = number heavy, Y ~ B(18, 0.2743)

P(Y ≥ 3) = 1 − P(Y ≤ 2) = 0.907

**Answer: (a) 0.274 (b) 0.907**

## 5 Projectile through a given point: two angles (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 35 m $s^{-1}$, at angle θ above the horizontal. It passes through the point P, 93 m horizontally and 6 m vertically from O.

Find the two possible values of tan θ, and the two possible values of θ to 1 d.p. Take g = 9.8 m $s^{-2}$.

Horizontal: $x = 35 \cos\theta \times t$. Vertical: $y = 35 \sin\theta \times t - 4.9t^2$

Eliminating t: $y = x \tan\theta - \frac{9.8x^2}{2u^2}(1 + \tan^2\theta)$

$\frac{9.8 \times 93^2}{2 \times 35^2} = 34.596$, so $6 = 93 \tan\theta - 34.596(1 + \tan^2\theta)$

$34.596 \tan^2\theta - 93 \tan\theta + 40.596 = 0$

$\tan\theta = \frac{17}{31}$ or $\frac{199}{93}$, so θ = 28.7° or 65.0°

**Answer: $\tan\theta = \frac{17}{31}$ or $\frac{199}{93}$; θ = 28.7° or 65.0°**

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## 6 Segments: exact area and perimeter (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB of a circle, centre O and radius 12 cm, subtends an angle of $\frac{\pi}{3}$ at O.

Find the exact area and the exact perimeter of the minor segment cut off by AB.

Sector OAB: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times \frac{\pi}{3}$; triangle OAB: $\frac{1}{2}r^2\sin\theta = 36\sqrt{3}$

Area = $24\pi - 36\sqrt{3}$

Chord AB = $2r\sin(\theta/2) = 12$

Arc AB = $r\theta = 4\pi$

Perimeter = arc + chord

**Answer: Area = $24\pi - 36\sqrt{3}$ cm$^2$, perimeter = $12 + 4\pi$ cm**

## 7 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first row of a theatre has 14 seats. Each row has 3 more seats than the row in front. Find

(a) the number of seats in row 52,

(b) the least number of rows for at least 1789 seats.

Arithmetic: a = 14, d = 3.

(a) $u_{52} = 14 + 51 \times 3 = 167$.

(b) $S_n = \frac{n}{2}(28 + 3n - 3) = \frac{n(3n + 25)}{2} \geq 1789$.

$S_{30} = 1725 < 1789$, $S_{31} = 1829 \geq 1789$.

**Answer: (a) 167 (b) 31 rows**

## 8 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 5 m and mass 4 kg rests on supports at C and D, where AC = 2 m and AD = 4 m, with R_C = $\frac{2}{3}$ R_D.

Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.

R_C = $\frac{2}{3}$ R_D. Moments about A: R_C × 2 + R_D × 4 = 4 g × AG

R_D($\frac{2}{3}$ × 2 + 4) = 4 g × AG with R_D(1 + $\frac{2}{3}$) = 4 g, so AG = $\dfrac{\frac{2}{3} \times 2 + 4}{\frac{5}{3}} = 3.2$

Tilting about D: R_C = 0. Moments about D: m g $(5 - 4) = 4$ g $(4 - 3.2)$

m = $\dfrac{4 \times 0.8}{1} = 3.2$

**Answer: (a) 3.2 m (b) 3.2**

## 9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - 2xy + 2y^2 = 72$.

Find the coordinates of the points on C where the tangent is parallel to the y-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\dfrac{2x + -2y}{-2x + 4y}$

Vertical tangent when $-2x + 4y = 0$, so $x = \frac{4y}{2}$

Substitute into the equation of C: $x^2 + (-2xy) + (2y^2) = 72$

Solve for the remaining coordinate

**Answer: $(-12, -6)$ and (12, 6)**

## 10 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−6, −7), B(−8, −5) and C(10, 1). Find its equation.

Centre is on the perpendicular bisector of each chord.

Bisector of AB: $-2x + 2y = 2$.

Bisector of BC: $18x + 6y = 6$.

Solving: centre (0, 1).

$r^2 = (-6 - 0)^2 + (-7 - 1)^2 = 100$.

**Answer: $(x + 0)^2 + (y - 1)^2 = 100$**