Day 188 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos 2\theta + 3\sin\theta - 2 = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - 3\sin\theta + 1 = 0$
$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = 1$

Principal values and symmetry in $[0, 2\pi)$

Answer: $\boldsymbol{\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$

2 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (2 kg) lies on a rough table with $\mu = 0.3$, and is joined by a light string over a smooth pulley (see diagram) to particle B (2 kg) on a smooth slope, $\alpha = 30°$.
Released from rest, the string is cut when B has moved 2.25 m. Find the further distance A slides before stopping.

A: $T - 0.3 \times 2 \times 9.8 = 2a$. B: $2 \times 9.8 \times \frac{1}{2} - T = 2a$

Adding: $9.8(2 \times \frac{1}{2} - 0.3 \times 2) = 4a$, so $a = 0.98$

$T = 2(0.98 + 0.3 \times 9.8) = 7.84$

When cut: $v^2 = 2 \times 0.98 \times 2.25 = 4.41$. Then A decelerates at $\mu g = 2.94$

Further distance $= \frac{4.41}{2 \times 2.94} = 0.75$

Answer: 0.75 m

3 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows $y = f(x)$, made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = 3f(-2x + 8) - 3$.

Inside f: replace x by $-2x + 8$, so x-coordinates become $(x - 8) \div (-2)$.
Outside: y-coordinates become $3 \times y - 3$.
A: $(7, -6)$ B: $(6, -15)$
C: $(5, -6)$ D: $(2, 0)$
Answer: A′(7, −6), B′(6, −15), C′(5, −6), D′(2, 0)

4 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O travelling at a constant speed of 14 m s^{-1}. 7 s later, car B leaves O from rest with constant acceleration 3 m s^{-2}, travelling in the same direction.
B overtakes A at the point X. Find (a) the time B takes to reach X, (b) the distance OX, (c) the speed of B at X.

Let B travel for t s. A: distance $= 14(t + 7)$; B: distance $= \frac{1}{2} \times 3 \times t^2$

Equate: $3t^2 - 28t - 196 = 0$, so $(t - 14)(3t + 14) = 0$
$t = 14$ (the other root is negative)

OX $= 3 \times 14^2 / 2 = 294$ m, and speed $= 3 \times 14 = 42$

Answer: (a) 14 s (b) 294 m (c) 42 m s^{-1}

5 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 recorded times (seconds) have mean 23 and standard deviation 3.
One value was recorded as 24 but should have been 16.
Find the correct mean and the correct standard deviation.

$\Sigma x = 12 \times 23 = 276$, so corrected $\Sigma x = 276 - 24 + 16 = 268$

Correct mean $= \frac{268}{12} = 22.33$

$\Sigma x^2 = 12(9 + 529) = 6456$, corrected $\Sigma x^2 = 6456 - 576 + 256 = 6136$

Variance $= \frac{6136}{12} - 22.333^2 = 12.556$, $\sigma = 3.543$

Answer: mean 22.3, standard deviation 3.54

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3 \sin t$, $y = 3 \cos 2t$.
Find a cartesian equation of the curve in the form $y = f(x)$.

$\cos 2t = 1 - 2\sin^2 t$

$\sin t = \frac{x}{3}$, so $y = 3(1 - 2 \times \frac{x^2}{9})$

Answer: $y = 3 - \frac{2}{3}x^2$

7 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, $P(A) = 0.4$, $P(B) = 0.25$ and $P(A \mid B) = 0.7$.
(a) Find $P(A \cap B)$.
(b) Find $P(A \mid B')$.

$P(A \cap B) = P(A \mid B) \times P(B) = 0.7 \times 0.25 = 0.175$
$P(A \cup B) = 0.4 + 0.25 - 0.175 = 0.475$
$P(B') = 0.75$
$P(A \cap B') = P(A) - P(A \cap B) = 0.225$

$P(A \mid B') = \frac{0.225}{0.75}$

Answer: (a) 0.175 (b) $\frac{3}{10}$

8 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 3n - 5$, $u_1 = 6$.
(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.
(b) Find the least n for which $u_n > 1022$.

$u_n = u_1 + \sum_{k=1}^{n-1}(3k - 5) = 6 + 3 \times \frac{(n-1)n}{2} - 5(n - 1).$

$= \frac{3}{2}n^2 - \frac{13}{2}n + 11.$

So $a = \frac{3}{2}$, $b = -\frac{13}{2}$, $c = 11$.

$u_{28} = 1005 \leq 1022$ and $u_{29} = 1084 > 1022$.

Answer: (a) $a = \frac{3}{2}$, $b = -\frac{13}{2}$, $c = 11$ (b) $n = 29$

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$\frac{dy}{dx} = e^{x-y}$, and $y = \ln 5$ when $x = 0$.

Show that $e^y = e^x + 4$ and hence find y in terms of x.

$e^y dy = e^x dx$, so $e^y = e^x + c$
$x = 0$: $5 = 1 + c$, so $c = 4$
Take natural logarithms

Answer: $y = \ln(e^x + 4)$

10 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{-3x + 17}{x - 3}$.

(a) Write down the equations of the asymptotes of C.
(b) The line $y = -2x + k$ is a tangent to C.
Find the possible values of k.

(a) Vertical: $x = 3$. As $x \to \infty$, $y \to -3$, so $y = -3$.
(b) $-3x + 17 = (-2x + k)(x - 3)$.
$-2x^2 + (k + 9)x + (-3k - 17) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 6k - 55 = 0$.
Answer: (a) $x = 3$, $y = -3$ (b) $k = -5$ or $k = 11$