Day 186 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.8, P(B) = 0.45 and P(A | B) = 0.85.
(a) Find P(A ∩ B).
(b) Find P(A | B′).

$P(A \cap B) = P(A \mid B) \times P(B) = 0.85 \times 0.45 = 0.3825$
$P(A \cup B) = 0.8 + 0.45 - 0.3825 = 0.8675$
$P(B') = 0.55$
$P(A \cap B') = P(A) - P(A \cap B) = 0.4175$
$P(A \mid B') = \frac{0.4175}{0.55}$

Answer: (a) 0.3825 (b) $\frac{167}{220}$

2 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population of bacteria is modelled by $P = P_0e^{kt}$, t in hours.
Initially P = 120 and after 6 hours P = 150.
Find k, and the time for the population to reach 360, to 3 s.f.

$P_0 = 120$. $120\,e^{6k} = 150$ so $e^{6k} = \frac{5}{4}$

$k = \frac{1}{6} \ln \frac{5}{4} = 0.0372$

$120\,e^{kt} = 360$ so $e^{kt} = 3$, $t = \frac{1}{k} \ln 3$

t = 29.5 hours
Answer: k = 0.0372, t = 29.5 hours

3 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{3x - 15}{x - 2}$.
(a) Write down the equations of the asymptotes of C.
(b) The line y = x + k is a tangent to C.
Find the possible values of k.

(a) Vertical: x = 2. As $x \to \infty$, $y \to 3$, so y = 3.
(b) $3x - 15 = (x + k)(x - 2)$.
$x^2 + (k - 5)x + (-2k + 15) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 2k - 35 = 0$.
Answer: (a) x = 2, y = 3 (b) k = −5 or k = 7

4 R-form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express 6 cos θ + 8 sin θ in the form R cos(θ − α), where R > 0 and 0 < α < 90°. Give α to 1 d.p.
(b) Hence solve 6 cos θ + 8 sin θ = −1 for 0 ≤ θ < 360°, giving answers to 1 d.p.

$R = \sqrt{6^2 + 8^2} = 10$ and $\tan \alpha = \frac{8}{6}$, so α = 53.1°

So $6 \cos \theta + 8 \sin \theta \equiv 10 \cos(\theta - 53.1°)$

$\cos(\theta - \alpha) = \frac{-1}{10}$

θ − α = ±95.7° (using α = 53.1°)
Adding or subtracting multiples of 360° to give 0 ≤ θ < 360°
Answer: (a) R = 10, α = 53.1° (b) θ = 148.9°, 317.4°

5 Relative motion of two projectiles (AQA, Edexcel, OCR, MEI, Eduqas)

Particles A and B are projected from a point O at the same instant, with initial velocities $(5\mathbf{i} + 24\mathbf{j})$ m s^{-1} and $(8\mathbf{i} + 20\mathbf{j})$ m s^{-1} respectively (i horizontal, j vertically upwards).
Both are still in flight after 2 s. Find (a) the distance AB at this time, (b) the angle AB makes with the horizontal, to 1 d.p.

Position of A: $5t\mathbf{i} + (24t - 4.9t^2)\mathbf{j}$; position of B: $8t\mathbf{i} + (20t - 4.9t^2)\mathbf{j}$
Subtracting, the $4.9t^2$ terms cancel: AB = $t(-3\mathbf{i} + 4\mathbf{j})$
After 2 s: AB = $(-6\mathbf{i} + 8\mathbf{j})$
Distance = $\sqrt{(6^2 + 8^2)} = 10$

The direction of AB is that of $(-3\mathbf{i} + 4\mathbf{j})$, which does not depend on t: tan angle = $\frac{4}{3}$

Answer: (a) 10 m (b) 53.1°

6 PMCC from summary data and a hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

For 25 pairs of values, $S_{xx} = 36$, $S_{yy} = 576$ and $S_{xy} = 92$.
(a) Calculate the product moment correlation coefficient r.
(b) The critical value for a one-tailed test at the 1% level with n = 25 is 0.4622. Carry out the test of H_0: $\rho = 0$ against H_1: $\rho > 0$.

$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}} = \frac{92}{\sqrt{(36 \times 576)}} = \frac{92}{144} = 0.6389$
H_0: $\rho = 0$, H_1: $\rho > 0$
$|r| = 0.6389 >$ the critical value 0.4622
Reject H_0: there is sufficient evidence of positive correlation between the variables
Answer: (a) r = 0.639 (b) reject H_0

7 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first row of a theatre has 14 seats. Each row has 3 more seats than the row in front. Find
(a) the number of seats in row 46,
(b) the least number of rows for at least 1219 seats.

Arithmetic: a = 14, d = 3.
(a) $u_{46} = 14 + 45 \times 3 = 149$.
(b) $S_n = \frac{n}{2}(28 + 3n - 3) = \frac{n(3n + 25)}{2} \geq 1219$.
$S_{24} = 1164 < 1219$, $S_{25} = 1250 \geq 1219$.
Answer: (a) 149 (b) 25 rows

8 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 140 N rests on a rough plane inclined at α to the horizontal, where tan α = $\frac{4}{3}$ and μ = 0.5.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

sin α = $\frac{4}{5}$, cos α = $\frac{3}{5}$. Normal reaction: R = 140 cos α + H sin α

About to slide down: H cos α + μR = 140 sin α, so $H(\frac{3}{5} + 0.5 \times \frac{4}{5}) = 140(\frac{4}{5} - 0.5 \times \frac{3}{5})$

H_min = 70

About to slide up: H cos α = 140 sin α + μR, so $H(\frac{3}{5} - 0.5 \times \frac{4}{5}) = 140(\frac{4}{5} + 0.5 \times \frac{3}{5})$

H_max = 770
Answer: $70 \leq H \leq 770$

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + xy + y^2 = 27$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + y}{x + 2y}$

Horizontal tangent when $2x + y = 0$, so $y = -\frac{2x}{1}$

Substitute into the equation of C: $x^2 + (xy) + (y^2) = 27$
Solve for the remaining coordinate
Answer: (3, −6) and (−3, 6)

10 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−10, 12) and B(−5, −13). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-10))^2 + 144 = (c - (-5))^2 + 169$.
Expand and solve: $10c = -50$, so $c = -5$.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|(-5) \times (-13) - 0 \times 12| = \frac{65}{2}$.

Answer: C(−5, 0); area $\frac{65}{2}$