Day 185 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A die is said to show heads with probability 0.25. In 20 trials it does so 2 times.
Test at the 10% level whether the probability is different from 0.25.

H_0: p = 0.25, H_1: p ≠ 0.25 (two-tailed). X ~ B(20, 0.25)
2 is below the mean 5, so use the lower tail: P(X ≤ 2) = 0.0913
Each tail is compared with 5%: 0.0913 is greater than 5%
Do not reject H_0: there is insufficient evidence that the probability is different from 0.25
Answer: Do not reject H_0 (p-value 0.0913 against 5%)

2 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−17, −5) and B(−5, 13). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-17))^2 + 25 = (c - (-5))^2 + 169$.
Expand and solve: 24c = −120, so c = −5.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
= $\frac{1}{2}|(-12) \times 13 - 0 \times (-5)| = 78$.
Answer: C(−5, 0); area 78

3 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The fill volume of a bottle is N(μ, 64) with known variance 64. The mean was claimed to be 46 ml.
A random sample of 9 has mean 50. Test at the 1% level whether the mean is greater than 46.

H_0: μ = 46, H_1: μ > 46. Under H_0, the sample mean ~ $N(46, \frac{64}{9})$

$$z = \frac{50 - 46}{8 \div \sqrt{9}} = 1.500$$

p-value = P(Z > 1.500) = 0.0668
0.0668 > 1%, so do not reject H_0
There is insufficient evidence at the 1% level that the mean has changed as claimed
Answer: Do not reject H_0 (z = 1.500, p-value 0.0668)

4 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 14 m s^{-1} from the top of a cliff. The top of the cliff is 52.5 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-52.5 = 14t - 4.9t^2$, so $4.9t^2 - 14t - 52.5 = 0$

$$t = \frac{14 + \sqrt{(14^2 + 19.6 \times 52.5)}}{9.8} = \frac{14 + 35}{9.8} = 5$$

Height risen above the cliff top = $\frac{14^2}{(2 \times 9.8)} = 10.0$ m
Total distance = 2 × 10.0 + 52.5
Answer: (a) 5 s (b) 72.5 m

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5 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 15 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially $\theta = 90$, and after 6 minutes $\theta = 65$.
Find A and k, then find when $\theta = 47$ (3 s.f.).

$t = 0$: $15 + A = 90$ so $A = 75$

$t = 6$: $15 + 75e^{-6k} = 65$ so $e^{-6k} = \frac{2}{3}$

$k = \frac{1}{6} \ln \frac{3}{2} = 0.0676$

$75\, e^{-kt} = 32$ so $t = \frac{1}{k} \ln \frac{75}{32}$

$t = 12.6$ minutes

Answer: A = 75, k = 0.0676, t = 12.6 minutes

6 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + ax^2 + bx + 40$, where a and b are constants.
$(x + 4)$ is a factor of f(x), and f(x) leaves remainder 40 when divided by $(x + 3)$.
Find a and b, and hence solve $f(x) = 0$.

$f(-4) = 0$: $-64 + 16a - 4b + 40 = 0$, so $16a - 4b - 24 = 0$.
$f(-3) = 40$: $9a - 3b + 13 = 40$.
Solving: $a = -3$, $b = -18$.
$f(x) = x^3 - 3x^2 - 18x + 40 = (x + 4)(x - 2)(x - 5)$
Answer: a = −3, b = −18; x = −4, 2, 5

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x^2\sqrt{5 - x}$, for $0 < x < 5$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{2x(20 - 5x)}{2\sqrt{5 - x}}$

$\frac{dy}{dx} = 0$: $20 - 5x = 0$, so $x = 4$

$y = 2 \times 4^2 \times 1$

Gradient changes from positive to negative: a maximum

Answer: (4, 32), maximum

8 Projectile through a given point: two angles (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 28 m s^{-1}, at angle θ above the horizontal. It passes through the point P, 32 m horizontally and 14 m vertically from O.
Find the two possible values of $\tan\theta$, and the two possible values of θ to 1 d.p. Take $g = 9.8$ m s^{-2}.

Horizontal: $x = 28 \cos\theta \times t$. Vertical: $y = 28 \sin\theta \times t - 4.9t^2$

Eliminating t: $y = x \tan\theta - \frac{9.8x^2}{2u^2}(1 + \tan^2\theta)$

$\frac{9.8 \times 32^2}{2 \times 28^2} = 6.4$, so $14 = 32 \tan\theta - 6.4(1 + \tan^2\theta)$

$6.4 \tan^2\theta - 32 \tan\theta + 20.4 = 0$

$\tan\theta = 0.75$ or 4.25, so $\theta = 36.9°$ or $76.8°$

Answer: tan θ = 0.75 or 4.25; θ = 36.9° or 76.8°

9 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(8, −1), B(1, 16) and C(−9, −8). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of AB: $-7x + 17y = 96$.
Bisector of BC: $-10x - 24y = -56$.
Solving: centre $(-4, 4)$.
$r^2 = (8 - (-4))^2 + (-1 - 4)^2 = 169$.
Answer: $(x + 4)^2 + (y - 4)^2 = 169$

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix}2\\2\\1\end{pmatrix} + s\begin{pmatrix}2\\-2\\0\end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix}7\\3\\4\end{pmatrix} + t\begin{pmatrix}3\\3\\3\end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $2 + 2s = 7 + 3t$; $2 - 2s = 3 + 3t$; $1 = 4 + 3t$
Solving the first two: $s = 1$, $t = -1$; the third equation is satisfied
Intersection: (4, 0, 1)
$\cos\varphi = \frac{|0|}{\sqrt{8}\sqrt{27}} = 0.0000$, $\varphi = 90.0°$
Answer: Lines meet at (4, 0, 1); angle 90.0°