Day 183 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos 2\theta + 3\cos\theta - 1 = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta + 3\cos\theta - 2 = 0$
$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -2$ (impossible, $|\cos\theta| \leq 1$)

Principal values and symmetry in $[0, 2\pi)$

Answer: $\boldsymbol{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

2 Normal approximation to a Poisson (CIE)

The number of customers arriving at a shop in one hour is Po(45).
Use a suitable approximation to find $P(X \geq 65)$, to 3 significant figures.

$\lambda = 45$ is large (greater than 15), so X is approximately N(45, 45)
Standard deviation $= \sqrt{45} = 6.708$
Continuity correction:
$P(X \geq 65) \approx P(Y > 64.5) = P(Z > 2.907) = 0.00183$
Answer: 0.00183

3 Time above a given height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards from ground level. It is more than 235.2 m above the ground for 2 s.
Find the speed of projection and the greatest height reached. Take $g = 9.8$ m s^{-2}.

By symmetry, the ball takes 1 s to rise from height 235.2 m to its highest point
Speed at height 235.2 m: $v = 9.8 \times 1 = 9.8$ m s^{-1}
Speed of projection: $u^2 = v^2 + 2gh = 96.04 + 2 \times 9.8 \times 235.2 = 4705.96$
$u = 68.6$. Greatest height $= \frac{u^2}{(2g)} = \frac{4705.96}{19.6} = 240.1$

Answer: 68.6 m s^{-1}, 240.1 m

4 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 recorded masses (grams) have mean 25 and standard deviation 2.
One value was recorded as 23 but should have been 25.
Find the correct mean and the correct standard deviation.

$\Sigma x = 20 \times 25 = 500$, so corrected $\Sigma x = 500 - 23 + 25 = 502$

Correct mean $= \frac{502}{20} = 25.10$

$\Sigma x^2 = 20(4 + 625) = 12580$, corrected $\Sigma x^2 = 12580 - 529 + 625 = 12676$

Variance $= \frac{12676}{20} - 25.100^2 = 3.7900$, $\sigma = 1.947$

Answer: mean 25.1, standard deviation 1.95

5 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where
$v = 3t^2 - 18t + 15$ for $0 \leq t \leq 6$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 6 seconds.

$v = 3(t - 1)(t - 5)$, so $v = 0$ when $t = 1$ and $t = 5$
$v > 0$ for $0 < t < 1$, $v < 0$ for $1 < t < 5$, $v > 0$ for $t > 5$: direction changes at each

$s = \int v\,dt = \frac{3}{3}t^3 - 9t^2 + 15t$

Distances: 0 to 1: 7; 1 to 5: 32; 5 to 6: 7
Total $= 7 + 32 + 7$
Answer: (a) t = 1, t = 5 (b) 46 m

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2y + xy^2 = 6$.
Find the equation of the normal to C at the point $(-3, 1)$,
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At $(-3, 1)$: $dy/dx = \frac{5}{3}$

Normal gradient $= -1 \div \frac{5}{3} = -\frac{3}{5}$

$y - 1 = (-\frac{3}{5})(x - -3)$, then clear fractions

Answer: $3x + 5y + 4 = 0$

7 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = x^2 + 1$. Also $fg(x) = -2x^2 + 5$, where f is
a linear function. (a) Find f(x).
(b) Solve $gf(x) = 82$.

(a) Let $f(x) = ax + b$. $fg(x) = a(x^2 + 1) + b = -2x^2 + 5$.
So $a = -2$ and $-2 + b = 5$, giving $b = 7$.
$f(x) = -2x + 7$.
(b) $gf(x) = (-2x + 7)^2 + 1 = 82$.
$(-2x + 7)^2 = 81$, so $-2x + 7 = \pm 9$.
$x = -1$ or $x = 8$.
Answer: (a) $f(x) = -2x + 7$ (b) $x = -1$ or $x = 8$

8 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 6| < -3x - 6$.

Boundary values: $x - 6 = -3x - 6$ gives $x = 0$;
$x - 6 = -(-3x - 6)$ gives $x = -6$.
Also the corner of $|x - 6|$ is at $x = 6$.
Testing each region (check the graph of each side) gives the solution set.
Answer: $x < -6$

9 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two boats P and Q move so that, t hours after noon, their position vectors in km are
$\mathbf{r}_P = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = 0$

$-10 + t(5) = 0$, so $t = 2$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, with length $\sqrt{5}$

Answer: $t = 2$, minimum distance $= \sqrt{5}$ km

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10 Binomial expansion with partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-5x + 5}{(1 + 2x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) in ascending powers of x up to x^3, stating the range of validity.

(a) $-5x + 5 \equiv A(1 - 3x) + B(1 + 2x)$: A = 3, B = 2.

$f(x) = \frac{3}{1 + 2x} + \frac{2}{1 - 3x}$.

(b) $\frac{1}{1 + cx} = 1 - cx + c^2x^2 - c^3x^3 + ...$ (use c = 2, then c = −3).

Coefficients of x^j: $3(-2)^j + 2(3)^j$.

Valid for $|x| < \frac{1}{3}$ (both expansions).

Answer: (a) A = 3, B = 2 (b) $5 + 30x^2 + 30x^3 + ...$; $|x| < \frac{1}{3}$