Day 180 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Least number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each battery is faulty with probability 0.12, independently.
Find the least number n of batterys that must be tested so that the probability of finding at least two faulty is greater than 0.95.

Let X ~ B(n, 0.12). $P(X \geq 2) = 1 - 0.88^n - n(0.12)(0.88)^{n-1}$
This has no simple algebraic solution, so test values of n
n = 37: $P(X \geq 2) = 0.9466$, which is not greater than 0.95
n = 38: $P(X \geq 2) = 0.9520$, which is greater than 0.95
$P(X \geq 2)$ increases with n, so the least value is 38
Answer: n = 38

2 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{2x + 1} = 2x - 1$.

Square both sides: $2x + 1 = (2x - 1)^2$.

$4x^2 - 6x = 0$, so x = 0 or $x = \frac{3}{2}$.

Check in $\sqrt{2x + 1} = 2x - 1$: $x = \frac{3}{2}$ gives $2 \geq 0$ (valid).

x = 0 gives right-hand side $-1 < 0$, so it is rejected.

Answer: $x = \frac{3}{2}$

3 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-8\mathbf{i} - 7\mathbf{j})$ N, $(2\mathbf{i} + k\mathbf{j})$ N and $(-6\mathbf{i} - 8\mathbf{j})$ N act on a particle of mass 4 kg. The resultant is parallel to $(-12\mathbf{i} - 7\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(-12)\mathbf{i} + (-15 + k)\mathbf{j}$

Parallel to $(-12\mathbf{i} - 7\mathbf{j})$: $\frac{-15 + k}{-12} = \frac{-7}{-12}$, so k = 8

R = $(-12\mathbf{i} - 7\mathbf{j})$ N, so $|R| = \sqrt{(-12^2 + -7^2)} = 13.89$
a = |R| ÷ 4
Answer: (a) k = 8 (b) 3.47 m s^{-2}

4 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\sin A = \frac{12}{13}$ and $\cos B = \frac{21}{29}$.
Find the exact values of tan(A + B) and sin(A − B).

$\cos A = \frac{5}{13}$, $\sin A = \frac{12}{13}$; $\sin B = \frac{20}{29}$, $\cos B = \frac{21}{29}$

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\frac{352}{377}}{-\frac{135}{377}} = -\frac{352}{135}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B = \frac{152}{377}$$

Answer: $\tan(A + B) = -\frac{352}{135}$, $\sin(A - B) = \frac{152}{377}$

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5 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 8 m and mass 50 kg rests on supports at C and D, where AC = 3 m and DB = 3 m. A girl of mass 50 kg stands on it at distance x m from A.

Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs $R_C \geq 0$ and $R_D \geq 0$. Limiting cases: $R_C = 0$ and $R_D = 0$ in turn

Tilting about D (girl beyond D): $50(1) = 50(x - 5)$, so $x = 5 + \frac{50 \times 1}{50} = 6$

Tilting about C (girl before C): $50(1) = 50(3 - x)$, so $x = 3 - \frac{50 \times 1}{50} = 2$

Answer: $2 \leq x \leq 6$

6 Without replacement: find the number of counters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 6 yellow counters and some white counters, n in total.

Two are taken without replacement. P(at least one yellow) = $\frac{17}{22}$. Find n.

P(neither yellow) = $\frac{(n-6)}{n} \times \frac{(n-7)}{(n-1)} = \frac{5}{22}$

$\frac{5}{22}\, n(n-1) = (n-6)(n-7)$

Rearranging gives a quadratic in n whose roots are n = 12 and one other value

The other root is $\frac{77}{17}$ so is rejected (not a valid number of counters)

n = 12

Answer: n = 12

7 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\pi/2} e^{2x} \sin x \, dx$

Let I be the integral. Apply parts twice (differentiating the trig function each time)

This returns a multiple of I, so $(1 + \frac{1}{4})I$ = boundary terms

$I = \frac{e^{2x}(2\sin x - \cos x)}{5}$

Answer: $\frac{1}{5} + \frac{2}{5}e^{\pi}$

8 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:

$x + y = -1$

$2x^2 + 2xy + y^2 = 37$

From the first equation $x = -1 - y$.

Substitute into the second equation and collect terms:

this gives $y^2 + 2y - 35 = 0$.

$y = -7$ or $y = 5$, giving $x = 6$ or $x = -6$.

Answer: (x, y) = (6, −7) and (−6, 5)

9 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{4}{17 + 6\sin\theta + 8\cos\theta}$

(a) Write $6\sin\theta + 8\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{6^2 + 8^2} = 10$, $\tan\alpha = \frac{8}{6}$, $\alpha = 53.1°$

Denominator $= 17 + 10\sin(\theta + 53.1°)$, which lies between 7 and 27

Greatest $f(\theta) = \frac{4}{7} = \frac{4}{7}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{4}{27} = \frac{4}{27}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 53.1° = 270°$, so $\theta = 216.9°$

Answer: (a) $10\sin(\theta + 53.1°)$ (b) greatest $\frac{4}{7}$, least $\frac{4}{27}$, $\theta = 216.9°$

10 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle with equation $(x - 2)^2 + (y + 3)^2 = 169$
passes through P(14, −8). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (2, −3). Gradient of radius CP $= -\frac{5}{12}$.

Tangent gradient $= \frac{12}{5}$: $12x - 5y = 208$.

$x = 0$: $y = -\frac{208}{5}$. $y = 0$: $x = \frac{52}{3}$.

Area $= \frac{1}{2} \times \frac{52}{3} \times \frac{208}{5}$.

Answer: $\frac{5408}{15}$