Day 178 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-7\mathbf{i} - 5\mathbf{j})$ N, $(0\mathbf{i} + k\mathbf{j})$ N and $(8\mathbf{i} - 5\mathbf{j})$ N act on a particle of mass 5 kg. The resultant is parallel to $(\mathbf{i} - 2\mathbf{j})$. Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(1)\mathbf{i} + (-10 + k)\mathbf{j}$

Parallel to $(\mathbf{i} - 2\mathbf{j})$: $\frac{-10 + k}{1} = \frac{-2}{1}$, so k = 8

R = $(\mathbf{i} - 2\mathbf{j})$ N, so |R| = $\sqrt{(1^2 + -2^2)} = 2.236$

a = |R| ÷ 5

Answer: (a) k = 8 (b) 0.447 m s^{-2}

2 Most likely value of a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, $\frac{1}{4}$).

(a) Show that P(X = r + 1) ÷ P(X = r) = $\frac{10 - r}{r + 1} \times \frac{1}{3}$.

(b) Find the most likely value of X and its probability (3 s.f.).

P(X = r + 1) ÷ P(X = r) = $\frac{C(10, r + 1)}{C(10, r)} \times \frac{p}{q} = \frac{10 - r}{r + 1} \times \frac{p}{q}$

With p = $\frac{1}{4}$ and q = $\frac{3}{4}$, p ÷ q = $\frac{1}{3}$

The ratio exceeds 1 when $(10 - r)\frac{1}{4} > (r + 1)\frac{3}{4}$, that is r < 1.75

So probabilities rise up to r = 2 and then fall; the mode is 2

P(X = 2) = 0.282

Answer: 2 with probability 0.282

3 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = t^2$, $y = 3t^3$.

Show that $y^2 = kx^3$ and find k.

$y^2 = 9t^6$.

$x^3 = t^6$.

So $y^2 = 9\,x^3$.

Answer: k = 9

4 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(4\mathbf{i} - 5\mathbf{j})$ m with velocity $(6\mathbf{i} - 8\mathbf{j})$ m s^{-1}. At t = 8, P has velocity $(-10\mathbf{i} + 16\mathbf{j})$ m s^{-1}.

Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i} - \mathbf{j})$, (c) the position vector of P at that time.

a = $\frac{v - u}{t} = \frac{(-10\mathbf{i} + 16\mathbf{j}) - (6\mathbf{i} - 8\mathbf{j})}{8} = (-2\mathbf{i} + 3\mathbf{j})$

v = $(6\mathbf{i} - 8\mathbf{j}) + t(-2\mathbf{i} + 3\mathbf{j})$

Parallel to $(\mathbf{i} - \mathbf{j})$: $(-2t + 6) \times -1 = (3t - 8) \times 1$, giving t = 2

r = $(4\mathbf{i} - 5\mathbf{j}) + 2(6\mathbf{i} - 8\mathbf{j}) + ½ \times 2^2\,(-2\mathbf{i} + 3\mathbf{j}) = (12\mathbf{i} - 15\mathbf{j})$

Answer: (a) $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-2} (b) t = 2 (c) $(12\mathbf{i} - 15\mathbf{j})$ m

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5 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(2\cos^{-1}(\frac{3}{5}))$.

Let $A = \cos^{-1}(\frac{3}{5})$: $\cos A = \frac{3}{5}$, $\sin A = \frac{4}{5}$, $\tan A = \frac{4}{3}$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{\frac{8}{3}}{-\frac{7}{9}}$

Answer: $\mathbf{-\frac{24}{7}}$

6 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.
Find (a) the total number of runners, (b) an estimate of the number with $11 \leq t < 37$.

Frequency = frequency density × width. Bar 10 to 14: height 7 squares, width 4, frequency 14
So one square of frequency density = 0.5 runners per second
Class frequencies = 25, 14, 18, 20, 18
Total = 95
Estimate = 0.5 × [7(3) + 3(12) + 5(8) + 6(3)] = 57.5
Answer: (a) 95 (b) 57.5

7 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 3\sqrt{x} + 2 = 0$.

Let $u = \sqrt{x}$: $u^2 - 3u + 2 = 0$.
$(u - 1)(u - 2) = 0$, so $\sqrt{x} = 1$ or $\sqrt{x} = 2$.
Square: $x = 1$ or $x = 4$.
Answer: $x = 1$ or $x = 4$

8 Optimisation with justification (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P is a point on the curve $y^2 = x$. Find the exact minimum distance from P to the point (3, 0), and the x coordinate of P at which it occurs.

$d^2 = (x - 3)^2 + x = x^2 - 5x + 9$

$d(d^2)/dx = 2x - 5 = 0$, so $x = \frac{5}{2}$

Second derivative is $2 > 0$: minimum

$d^2 = \frac{11}{4}$

Answer: distance $= \mathbf{\frac{\sqrt{11}}{2}}$**, x** $= \mathbf{\frac{5}{2}}$

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9 Binomial expansion with partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-24x + 2}{(1 - 3x)(1 + 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) in ascending powers of x^3, stating the range of validity.

(a) $-24x + 2 \equiv A(1 + 3x) + B(1 - 3x)$: $A = -3$, $B = 5$.

$f(x) = \frac{-3}{1 - 3x} + \frac{5}{1 + 3x}$.

(b) $\frac{1}{1 + cx} = 1 - cx + c^2x^2 - c^3x^3 + \ldots$ (use $c = -3$, then $c = 3$).

Coefficients of x^j: $-3(3)^j + 5(-3)^j$.

Valid for $|x| < \frac{1}{3}$ (both expansions).

Answer: (a) $A = -3$, $B = 5$ (b) $2 - 24x + 18x^2 - 216x^3 + \ldots$; $|x| < \frac{1}{3}$

10 Area between a curve and a line (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For $0 \le x \le \pi$, the curves $y = \sin x$ and $y = \sin 2x$ meet at the origin and at $x = \frac{\pi}{3}$. Find the exact area enclosed between them for $0 \le x \le \frac{\pi}{3}$.

For $0 < x < \frac{\pi}{3}$, $\sin 2x > \sin x$

Area $= \int_0^{\pi/3} (\sin 2x - \sin x)\,dx = \left[-\frac{\cos 2x}{2} + \cos x\right]$

$= \left[\left(-\cos\frac{2\pi}{3}/2 + \cos\frac{\pi}{3}\right) - \left(-\frac{1}{2} + 1\right)\right] = \left[\frac{3}{4} - \frac{1}{2}\right]$

Answer: $\frac{1}{4}$